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ABSTRACT

This document contains the proceedings from a "Math and Science for Girls" symposium. The conference was aimed at identifying teaching strategies that are particularly effective for girls. The book is divided into three parts: research reports, small group workshops, and miscellaneous items. The research reports are "Women in Science: Where Do We Go From Here?" (Jane Butler Kahle); "Warming Up the Chilly Climate" (Bernice Resnick Sandler); "Women's Learning Styles and the Teaching of Mathematics" (Judith Jacobs); "Panel on Recent Research and Recommendations of Professional Organizations" (Shirley Malcom, Leslie Hiles Paoletti, and Rena F. Subotnik); and "Is Gender an Issue in Math Class? Perhaps It's Time to Change the Subject" (Joan Countryman). Workshop titles are: "Women's Metaphors for Math" (Dorothy Buerk); "Gender and Technology (Cornelia Brunner and Margaret Honey); "Cooperative Small Groups" (Carolyn Hopp); "Mathematics in a Cultural and Historical Context" (Claudia Henrion); "The Math, Science and Engineering Major in College" (Barbara Lazarus); and "Science Teaching and Diverse Learning Styles" (Jan Serie). The final section contains questions and reactions generated by participants, a follow-up questionnaire, and a participant list. (MKR)

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MATH & SCIENCE FOR GIRLS

A Symposium Sponsored by The National Coalition of Girls' Schools

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MATH & SCIENCE FOR GIRLS

A Symposium Sponsored by

THE NATIONAL COALITION OF GIRLS' SCHOOLS

WITH SUPPORT FROM THE KLINGENSTEIN FUND

THE COMPLETE PROCEEDINGS

Foreword

THE WORDS COOPERATION, COLLABORATION, AND connection appear throughout this book as descriptors of how girls are best served in the disciplines of math and science. They also aptly describe the inspiration, design, development, and production processes that have brought this book from dream to reality. Ann Pollina, Dean of the Faculty at Westover School, and Louise Gould, Dean of the Faculty at Ethel Walker School, had a vision of bringing together classroom teachers and national researchers during a symposium on math and science for girls. They also hoped for a book that would chronicle all that was offered and shared during the symposium.

The pathway from vision to reality was long and full of hard work. Sharyn Moody transcribed all of the tapes of the symposium sessions. Molly Laird and Alexandra McClennen edited and critiqued the evolving manuscript. Sara White of Abboud & White Design used her writing and design skills in ways that gave an appropriate visual style to the substance of the text. We extend heartfelt thanks to each of these talented and dedicated women. Without them, this project would not have taken its present form.

Finally, we thank the John and Esther Klingenstein Fund and the member schools of the National Coalition of Girls' Schools for believing in and supporting the *Math and Science for Girls Symposium*. This endeavor has brought an appropriate and overdue focus on teaching and learning strategies for girls in math and science. May you enjoy and learn from this critically important work. ❖

WHITNEY RANSOME
MEG MILNE MOULTON
Executive Directors
THE NATIONAL COALITION OF GIRLS' SCHOOLS
Concord, Massachusetts
November, 1992

Introduction

IN THE LAST FIVE YEARS numerous and varied professional organizations have published major guidelines for revising mathematics and science education. In most reports a prominent focus has been the need for increased participation of under-represented populations in the talent pool, especially women. What teaching strategies and methodologies will best enable young women to succeed in mathematics and science careers?

The recent research on women, particularly that of Carol Gilligan and Nona Lyons (*In a Different Voice* and *Making Connections*) and Mary Belenkey, Blythe Clinchy, Jill Tarule, and Nancy Goldberger (*Women's Ways of Knowing*), documents that women have different ways of establishing values and substantially different styles of learning than do men. More and more, research must focus on the successes of women when they are allowed to establish the learning climate.

To meet the national challenge of inspiring young women to continue the study of mathematics and science, girls' schools — an untapped resource — should be used as laboratories for investigation of how girls learn best. Recognizing that fact, the National Coalition of Girls' Schools, with support from the Klingenstein Fund, sponsored **Math and Science for Girls: A Symposium** at Wellesley College. From June 16–20, 1991, 82 teachers, representing 46 independent schools from California to Virginia and Ontario to Texas, converged to share their experience in teaching mathematics and science to girls. The conference was aimed toward identifying teaching strategies that are particularly effective for girls. It examined the culture underlying these strategies in the hope of generating gender balanced methods of math and science teaching that can be imported to any school environment.

The symposium moved in three distinct phases. In the first, speakers of national repute engaged in gender research reviewed their findings with participants. Second, participants visited the sites of various mathematics and science intervention programs for girls. The third, and most important phase of the conference, made use of the most undervalued resource in the struggle for gender equity—the classroom teacher. A day of small group workshops with educators selected for their innovation and experience was followed by two sessions of brainstorming and exchange. Grouped into task forces by discipline, participants were asked to generate descriptions of specific classroom strategies that are successful for girls. The outcome was the Task Force Reports, which has been published in a separate booklet available through the National Coalition of Girls' Schools.

What follows are the speeches, workshop presentations, prepared papers, selected conversations, and major findings from the symposium. Some of the material assembled comes from the taped transcripts of workshop offerings and presenter-participant exchanges. For this reason, the text may appear at times disjointed. These are the sections that should be seen as nuggets of wisdom and insight rather than complete concepts. ❖

CONFERENCE ORGANIZERS

ANN POLLINA
Dean of the Faculty
Westover School

LOUISE GOULD
Dean of the Faculty
Ethel Walker School

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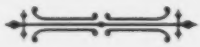

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PART ONE

Reports of the Research

Jane Butler Kahle

EDITOR OF "WOMEN IN SCIENCE"; MIAMI UNIVERSITY OF OHIO

DR. JANE BUTLER KAHLE, the Condit Distinguished Professor of Science Education and Professor of Zoology at Miami University, has worked extensively in the preparation of biology teachers as well as in the assessment of the special needs and issues surrounding women and minorities in science. Her previous appointments include: Professor of Biological Sciences and Education and Associate Dean of the Graduate School at Purdue University, and Dean of the College of Education at University of Northern Colorado.

Internationally, Dr. Kahle has evaluated science education programs and projects in the Philippines, Thailand, Indonesia, Australia, China, New Zealand, Britain, Norway, Sweden, Denmark, Israel, and the United Arab Emirates. She has been a member and convenor of the International Committee of the Gender and Science and Technology Association and currently is Chairman of its Board of Directors. Nationally, she has served as president of the National Biology Teachers Association and the National Association for Research in Science Teaching. She has chaired the Board of Directors of the Biological Sciences Curriculum Study (BSCS) and Section Q, American Association for the Advancement of Science (AAAS). Currently, she serves as Chairperson for the National Science Foundation's Committee on Equal Opportunities in Science and Engineering, and for its Advisory Committee in Preservice Teacher Education. She is also a member of the AAAS Committee on Opportunities in Science. She serves as an advisor/consultant for projects at the National Academy of Science, the National Science Teachers Association, the AAAS, the Association of American Colleges, the American Association of University Women Foundation, and several universities.

Dr. Kahle is the author of over 80 articles, four monographs, ten book chapters, and six books. She has served as a reviewer for five scholarly journals and directed more than 16 funded projects. ❖

Women in Science

~ *Where do we go from here?* ~

Jane Butler Kahle

NATIONAL DATA INDICATE THAT the gains that women have made in the last two decades, concerning their entrance and retention into the scientific and technological pipeline, are eroding. The numbers of women entering certain fields (notably engineering and statistics) leveled in 1984 and have begun to decline. Furthermore, in fields that women have entered in high numbers (biology and psychology) a solid, but unacknowledged, glass ceiling affects their ability as a group to contribute to their fullest extent. The percentage of women who are under- or unemployed in technical fields has remained at a level consistently higher than the percentage for men. These findings are in spite of two decades of intervention programs designed to increase the numbers of girls and women studying for careers in mathematics, science, and engineering, or the presence of excellent science and math courses in private girls' schools. Therefore we must ask ourselves two questions: First, *what has slowed our progress*, and, second, *where do we go from here*? I shall postulate that today the situation is different for girls in math than it is in science and that those differences have resulted in more progress in math than in science. Then I shall discuss directions that will help to guide further work and to determine *where do we go from here*.

Math and Girls

Because competency in mathematics is a prerequisite for entrance and persistence in scientific and technical fields, the study of mathematics was defined early as the "critical filter" (Sells, 1976). Furthermore, studies in the 1970s showed that differences in SAT-M scores between boys and girls could be partly attributed to different enrollment patterns in high school mathematics. Therefore, initial efforts to encourage girls in science and engineering focused on mathematics. Colleagues at Lawrence Hall of Science were very successful in identifying barriers to girls, including high math anxiety, low parental expectations, and competitive classroom climates. The outcome of the early intervention programs was that more girls began to enroll

in algebra, geometry, and trigonometry. Furthermore, as girls began taking more math courses, the gender gap in math achievement began to close (Linn & Hyde, 1989). Today there are large disparities in the numbers of girls and boys only in pre-calculus and calculus courses, and the achievement gap persists only on the SAT-M test that, according to some researchers, focuses on "mathematics fluency" or "glibness" (Linn, 1990).

Recently there have been interesting contrasts between girls' attitudinal, interest, and self-confidence patterns toward math and science. Data from the National Assessment of Education Progress Survey of Math provide a national perspective. Although 9-year-old boys and girls score the same at all proficiency levels in math, disparities begin to occur by age 13 and have increased by age 17 (Dossey, et al., 1988). However, there is little or no decline in girls' perceptions of the value of a career in mathematics (stable at 40%), or in the usefulness of math (73% to 70%) between ages 13 and 17. As I will discuss later, both the value that children assign to a subject and their perceived interest in a subject affect their later decisions concerning choice of subjects.

Another difference between math and science is the teacher's own perception of the value and worth of the subject in the student's life. On the whole, elementary teachers believe that math is a *basic*; and that it has value in a myriad of life roles. Science, on the other hand, is perceived by many elementary teachers as a supplemental subject and one that may not be needed by many children in their adult roles. Eccles' work (1989) demonstrates that many parents value math and science education differently as well.

The two decades of intervention programs in math were able to use and to build upon the positive attitudes of elementary teachers and parents toward mathematics. In addition, workshops and programs that were aimed at reducing math anxiety, increasing self-confidence, and improving enrollment of girls also included strategies for doing mathematics. That is, both computational and problem-solving skills were included. However, a different pattern developed in the workshops, projects, and research that focused on girls in science during the early 1980s.

Science and Girls

Several years after the initial intervention programs in math, we began to examine the disparities in achievement and interest in science expressed by girls and boys. The work was instigated by the findings of the 1976-77 National Assessment of Educational Progress' second Survey of Science. Although girls' achievement levels trailed those of boys at all levels, explanations for the differences were found among their responses to other types of questions. For the first time, questions concerning children's attitudes toward science and scientists, their desire to work in a scientific field, and their opinions of science classes were included in the survey. Responses

indicated that the gender issues in science were more complicated than they were in math, where achievement differences could be traced to number and type of courses taken and where few attitudinal differences were identified among young children. In science, attitudinal differences were evident by age 13 and the gender gap in achievement widened from age 9 to 13, although most boys and girls are enrolled in similar science classes during those years. For example, in the third grade both girls (67%) and boys (66%) respond that they think what they learn in science class is useful in everyday life. In 7th grade, both boys' and girls' responses continue to be fairly high (54% and 57%). However, boys retain that positive attitude through senior high school, while girls' perceptions of the utility of science fall by 11% during those years. The same is true of students' responses concerning an interest in a career in science. Boys and girls respond the same in the 7th grade, but girls' responses fall below those of boys by the 11th grade. The deterioration in attitudes about, and interest in, science by girls is paralleled by a decline in their performance. It may be documented that girls simply stop doing science, particularly physical science, during middle/junior high school. For example, during those years the number of girls who have "tried to fix something mechanical" declines, while the number of boys doing those activities rises dramatically (Mullis & Jenkins, 1988). In science, 9-year-old boys and girls score approximately the same at all four proficiency levels, but by age 13, girls fall behind boys at the upper two levels (*ability to analyze scientific procedures and data*; *ability to integrate specialized scientific information*), and they continue to lag behind boys through high school (Mullis & Jenkins, 1988). Unfortunately, the gender gap in achievement has doubled in the last decade.

Two seminal studies teased out explanations for the differences in achievement and attitude. First, Kahle and Lakes (1983) showed that 9-year-old girls expressed desires similar to those of boys when they were queried about what science activities they *would like to do*. However, when asked what science they *had done*, responses showed that boys, compared with girls, had had more opportunities to use scientific equipment, to perform science experiments, and to take science-related field trips. In the second study, Linn and others analyzed the response patterns for achievement items and found that at all age levels tested (9, 13, and 17), girls selected the "*I don't know*" response significantly more often than boys. (Linn, et al., 1987). Whereas boys were willing to risk guessing, girls were less willing to take a chance. That difference in selection pattern accounted for a considerable amount of the achievement difference between girls and boys. In the decade since NAEP's second Survey of Science there have been many intervention projects, designed to attract and to retain young women in science. However, today the numbers of girls and women indicating a desire and showing a persistence to study science are declining. *What are the causes for the decline* and *how do we stop the erosion?*

NATURE OR NURTURE?

Gender is what a culture makes of sex—it is the cultural transformation of male and female infants into adult men and women (Keller, 1986, p. 122). The concern about sex versus gender-based differences in science and mathematics is more than a semantic issue. The use of *sex* or *gender* differentiates between a biological or environmental origin for any given trait, behavior, or aptitude. For years scholars and practitioners have argued whether the observed differences in interest and achievement of boys and girls in science and mathematics were inherent or were the result of acculturation. That is, were they caused by *nature* or *nurture*? Were they related to a child's sex or a result of one's gender?

Basically, the biological argument maintains that any sex differences in science aptitudes or abilities are due to differences in genetic make-up, in hormonal levels at puberty, or in developmental patterns of boys and girls. Kahle and Danzl-Tauer (1991) have shown that the scientific evidence for genetic, hormonal, or developmental factors contributing to achievement differences in science and math is weak or non-existent. Therefore, researchers have tried to identify environmental factors that contribute to gender-based differences in science and math.

Environmental Factors

Almost a decade ago, I began to identify factors affecting the achievement and retention levels of girls and women in science. I considered three general areas: educational, socio-cultural, and personality. Educational included factors associated with formal and informal learning, while the socio-cultural area included all aspects of the culture and society in which a child lived. Personality included preferences in learning styles as well as differences in attribution patterns. Because I did not consider those attributes to be biologically based, I used the psychological term "personality" rather than the more value-laden terms of ability and aptitude. My research focused on educational factors, and a decade of research and intervention projects revealed specific teaching behaviors and instructional techniques that encouraged girls as well as boys to continue in science. Figure 1 delineated nine behaviors and techniques found among biology teachers who successfully encouraged many girls to continue in science. In addition, classroom climates and school organizational factors that fostered equity in the corridors, gymnasiums, and counseling offices were identified (Kahle, 1985; 1987, Kahle, Anderson & Damnjanovic, 1991).

During the same time period, a four-year action research project was conducted in eleven schools in Manchester, U.K. Children were followed for three years prior to making their subject choices in upper secondary school (Whyte, 1986). Analyses suggested promising instructional and curricular

practices. However, results were mixed and inconclusive across the various schools. Partly as a result of that study, Alison Kelly, one of the principal investigators, identified environmental factors that affect girls' and women's decisions to study and to participate in science (Kelly, 1981). Her thesis was that science as an intellectual domain is perceived as masculine and that this perception discourages girls from expressing interest in science, from achieving well in science, and from continuing to study science. She noted three ways in which science is perceived as masculine. They included: the numbers who practice and who are rewarded in science; the way in which science is packaged in curricular and instructional materials; the way in which science is practiced in schools, including student-teacher and student-student interactions. Although Kelly mentioned that the masculinity of science might have a biological basis, she concluded that there was inconclusive evidence for one. Instead, she described how the overwhelming number of men who both do and teach science combine with the way in which science is packaged and practiced to produce a masculine image. That image, she argued, is reproduced in schools where it discourages girls from active participation in science. All three of Kelly's factors are environmental in origin, and intervention projects, including those in single sex schools, have focused on changing the practice and packaging of science and mathematics in order to affect the numbers who do and teach science (Kahle, 1987; Jorde & Lea, 1987; Rennie, Parker & Hutchinson, 1985). The continuum of research that investigated factors affecting girls in mathematics and science courses is shown in Figure 2.

Research from the fields of education, sociology, and psychology continues to provide evidence that environmental, not biological, factors affect the achievement and retention levels of girls in science. For example, Alice Eagly (1987) hypothesizes that sex differences are the product of social roles that regulate behavior in adult life. According to social-role theory, gender differences in math and science are attributable to different gender-role expectations which encourage conformity to specific roles and behaviors. Girls and boys differ in their attitudes about and interest in science and mathematics because they accept different gender-role expectations.

Eccles' work (1989) also supports the environmental argument. As a result of a large study, involving thousands of mid-western girls and their parents, Eccles has concluded that environmental, not biological, factors are responsible for the different achievement and interest levels of girls and boys in math. She delineates four types of environmental factors: first, causal attribution patterns for success and failure; second, the input of socializers; third, gender-role stereotypes; and fourth, individual perceptions of various possible tasks. Eccles argues that, unless positive intervention occurs, the combined effect of those four types of factors will lead inevitably to lowered interest and achievement levels in math and science for girls, when compared to boys. For example, all four factors affect a child's perceived value of an activity. If an activity involves math or science, girls tend to undervalue it. In

addition, the four factors affect a child's expectation for future success. Girls', as opposed to boys', expectations for success in math and science decline as they progress through school. That decline occurs in spite of the fact that girls' grades do not show a similar demise. According to Eccles, both one's perceived value of an activity and one's expectation for success in it influence the amount of effort one expends on the activity, the performance level that can be achieved, and the decision to participate in, or to continue, the activity. Eccles has found that when asked to choose between advanced math and English courses, significantly more high school girls, compared to boys, select English regardless of their academic grades in the two subjects. As Eccles states,

> As was true with confidence in their abilities, the females showed a linear decrease over age in their assessment of the value of mathematics. This pattern was not evident in the male population as the males moved through these secondary school years. The confidence pattern was further paralleled by a growing discrepancy between the value that the females attached to mathematics and English. Again, in contrast, the males' responses were about equal for the two subjects. As the females moved through high school, they reported valuing English increasingly more than they valued mathematics (Eccles, 1989, p. 43).

More recently, researchers have looked at changes in girls' levels of self-confidence as they mature. Gilligan and others have showed that, in general, girls' levels of self-confidence decline in adolescence (Prose, 1990). The recent American Association of University Women study demonstrates that girls' self-confidence in science and math drops precipitously in adolescence (AAUW, 1990). Girls' decline in confidence in science and math is exacerbated by the masculine images of both subjects, many girls devalue those subjects during the critical early adolescent years.

Biological Factors

Although the biological argument has largely been put to rest, a last remaining vestige concerns whether differences in spatial skills are genetic or have an environmental basis. Understanding and interpreting the effect of spatial skills has been clouded by both imprecise and inadequate research (Tartre, 1990). Simply put, spatial skills involve the ability to perceive and to manipulate spatial relationships. Although sex differences have been found, spatial skill differences within one sex are greater than those between girls and boys. In an attempt to determine the effect of spatial skills, Linn and Peterson (1985) did a meta-analysis of studies concerned with any one of three defined types of spatial skills. They found that there were no significant

differences between boys and girls in the skill referred to as spatial visualization. However, they found slight but significant differences in the scores of boys and girls (favoring boys) in the skills of spatial orientation and spatial perception.

Although differences have been found, researchers do not agree that, or how, spatial skills relate to achievement in math and science (Tartre, 1990). Furthermore, two intervention projects have indicated that spatial skill development is heavily influenced by environmental factors; that is, it is a learned attribute. In the Girls into Science and Technology (GIST) project, Kelly and her colleagues found that after one semester in a technical trades course (industrial arts) girls and boys scored equally well on a test of spatial skills (Whyte, 1986). More recently, Glenda Lappan and her colleagues have developed and tested a new math curriculum that enhances children's spatial skills (Ben-Chaim, et al., 1988). Although gender-differences are found prior to studying the curriculum, no significant differences are found between the scores of boys and girls on tests of spatial skills afterwards.

Environmental, not biological, factors affect both the achievement levels and the differences in girls' and boys' expressed interest in science. Many of the identified factors suggest simple remedies.

Directions for Improvement

Schools provide the most important place to make a positive impact on the self-esteem of adolescents, as well as on their career preferences. The relationships between math and science, self-esteem, and career preferences provide an important example of the kind of difference schools can make. Because of the differences in the ways adolescent women and men think about math and science, the influence of teachers on young women and their self-esteem and careers is particularly strong (AAUW, 1991, p. 15).

Both intervention programs and research studies indicate directions for improving the entrance, achievement, and retention of girls in science as well as in math courses. Briefly, those directions include: the improvement of girls' self-confidence and self-perceptions of ability in science as well as in math; the development of the skills for doing math and science for all youngsters; the emphasis on teaching strategies that promote equitable education; and the demasculization and demystification of science.

Educational Factors

One demonstrated way to improve a girl's self-confidence about becoming a scientist is through active intervention with elementary teachers. A project, completed in 1983 in Perth, Australia, by Rennie, Parker, and Hutchinson (1985), was replicated recently in a suburb of Cincinnati (Kahle, et al., 1991). Both projects provided training in the skills for doing science

or in equitable teaching strategies to matched groups of fourth and fifth grade teachers. Initial questionnaires, completed by both students and teachers, showed that topics in physical science were less often taught and more often perceived as appropriate for boys in Australia and the U.S. The teacher workshops were short and involved one of three types of training: science skills, equity and skills, or equity only. Important changes occurred in children's perceptions of their ability to be scientists.

In both countries, the workshops in the skills of doing science stressed hands-on science and manipulation of equipment. The equity workshops discussed issues in gender equity and cooperative group learning, as well as basic skills of science such as observing phenomena, measuring change, transforming data, and communicating results. Because of the gender-role stereotyping of physical science as more appropriate for boys, both projects used electricity for the science lessons. The important changes in attitudes occurred in spite of a "boy friendly" topic.

In Australia, significantly more girls (85%) in the classes taught by teachers in the equity workshop, compared to girls (70%) in the classes taught by skills workshop teachers, indicated that they felt they could be scientists. Furthermore, ninety percent of boys in both groups answered positively. In discussing their findings, the researchers concluded that the results indicated the fallacy in the popular notion that girls' attitudes as well as achievement levels in science are improved by selecting topics and subjects in which girls indicate an interest. As they state,

> If the teachers... had based their programs on students' expressed or expected interests, then the girls concerned would never have experienced working with electricity, and would never have been given the opportunity to find out whether they had a liking or an aptitude for the area (Parker, 1985, p. 16).

Findings from the American replication of the study suggest that today teachers hold fewer stereotypic views about the appropriateness of selected science topics for girls and boys. However, this more controlled study (all groups received equal length workshops and a control group was added) resulted in dramatically improved levels of self-confidence by both boys and girls. Responses from students whose teachers had participated in one of three workshops, skills, equity and skills,or equity, showed increased self-confidence. When queried as to whether they felt they could be a scientist before and after their teachers received workshop training, both girls and boys responded positively. However, the remarkable gains by girls, particularly by those whose teachers were in the equity workshop, testifies to the effectiveness of equity training.

Other intervention programs and projects, including my own work in the U.S. and Kelly's in the U.K., focused primarily on improving girls' self-

confidence by introducing topics or curriculum considered to be "girl-friendly." Such topics were mainly biological and often involved societal issues. Seldom was the quantification of science stressed nor were specific skills for doing science emphasized. However, a small field-test in the rural mid-west provided an unplanned opportunity to test whether "girl-friendly" curriculum and materials or skill training was more important in keeping girls in science classes.

During a study of factors affecting rural girls' continuation in high school science courses, I noted that course grades continued to form the basis of girls' perceptions of their ability in science in spite of a year-long intervention program stressing the skills of doing science as well as increased laboratories and discussion activities. However, although self-perceptions of ability were not affected by the program, self-reported levels of confidence were. For example, more girls in the intervention program, as compared to those in a control group, indicated that women could be scientists. More importantly, significantly more of them opted to take chemistry, an elective course. Clearly, their confidence in doing science was positively affected by intervention activities that combined the skills in doing science with equitable teaching strategies.

Changing children's self-perceptions of their ability in science is an important direction for improvement. The importance of self-perception of ability has been demonstrated also by a study by Licht and her colleagues (Licht, et al., 1989). They found that girls may begin a lesson or activity with lower expectations for success than boys do, although the girls' lower expectations are unrealistic in light of their actual performances. All of the fifth graders in Licht's study were asked to indicate their perceived level of smartness in four subjects. Both boys and girls ranked their smartness in subjects in the following order: math, reading, social studies, and science. There was no difference in the self-perceptions of boys and girls in math and reading, but significant differences were found between boys' and girls' self-perceptions of smartness in social studies ($p < .025$) and science ($p < .01$). Licht and her colleagues attribute at least part of that difference to the type of feedback received. The researchers found that teachers gave both more and less ambiguous feedback to the students in math and reading than in the other two subjects and that boys reported receiving more overall feedback than did girls (2.76 vs. 2.38). Because girls more often than boys attribute failure in a task to low ability, the lack of, or ambiguity in, feedback differentially affected girls. Girls' loss of self-confidence in science, as opposed to math, may be related to the different amount and type of feedback provided in the two subjects by most elementary teachers. Clearly, teachers can change the type of feedback provided and that change will affect girls' perceptions of their ability in science as well as in math.

Research also indicates other teaching strategies that will improve girls' achievement and retention levels in science. Most science and math instruc-

tion involves whole-class activities and during those sessions certain types of students are favored. Tobin and others have called those students "target students" (Tobin, 1987; Tobin and Gallagher, 1987; Tobin, Kahle, and Fraser, 1990). Briefly, there are two types of target students: 1) those who are most able, and 2) those who are high risk takers. Tobin argues that knowledge is transmitted by teachers to target students and through them to the class as a whole. This is done by teachers addressing most questions to target students, by target students volunteering more frequently, and by teachers elaborating on target student responses more frequently and at greater length than they do for other students. He maintains that most students, therefore, develop understandings in a relatively covert way. The prevalence of target students in classrooms, including those in math and science, is tied to a reward structure that encourages competition rather than one that emphasizes learning. In co-educational classrooms male to female ratios of target students are 4:1. However, Tobin and his colleagues have found that both types of target students are present in girls' schools and that their effect on classroom interactions is similar to that found in co-educational situations.

In all educational settings the presence and use of target students disadvantages many learners. For example, lower level cognitive questions are directed to as many as two-thirds of the class, while almost all higher level questions are directed to a few target students. Furthermore, in grades nine through twelve, target students tend to be constant year after year. That finding indicates that students have cumulative differential experiences even when they enroll in the same course options and that those differences occur in both single-sex and co-ed schools.

A ten-week observation of two Australian teachers indicated that even well-meaning teachers, who do not hold sex-role stereotyped views may have target students (Kahle, 1990). A study of two teachers, Peter and Sandra, suggested that Peter held firm, sex-role stereotyped beliefs about the appropriateness of science for girls and boys. In his classroom, he called on girls more frequently than he did boys, but he asked higher-order cognitive questions almost exclusively of boys. Although he rewarded girls with more attention, his focus was on their personal, rather than their intellectual, traits. Peter insisted that he favored girls in his teaching, and outwardly he did. He assigned them, on the average, higher grades; he called on girls more frequently than he did boys; and he named equal numbers of boys and girls when asked to identify bright students. However, the message was clear in Peter's class that success in science was unusual for a girl and usual for a boy. Therefore, in spite of receiving his attention and appreciation, few girls, compared to boys, continued to study science. Sandra, on the other hand, served as a role-model for the girls in her class. She was both knowledgeable and concerned about equity issues. Yet, as a management strategy, she used many male target students, and she adjusted seating arrangements and

laboratory groups in ways that disadvantaged girls in order to control disruptive boys. Only by long and careful analysis of Sandra's class were the subtle gender differences apparent. Helping Sandra learn how to become a teacher/researcher enabled her to evaluate her own teaching and to create the equitable classroom she desired.

One promising direction is to assist teachers in becoming researchers in their own classrooms. Basic ethnographic skills in classroom observation can assess the presence and use of target students. Once the problem is identified, teachers can more easily monitor teacher-student interactions in order to involve more students in higher-level cognitive discourse. Teachers can work in pairs to observe each other; single teachers may audio-tape their lessons or ask students to record the classroom interaction patterns.

Socio-cultural Factors

Changing educational factors can have a profound effect on the self-confidence as well as the ability of girls to do science. However, changing the broader socio-cultural climate in which women work and live is another matter. Eccles' work clearly shows the effect of parental expectations, which change for girls as they grow older. Although parents expect both their sons and daughters to do well in math in elementary school, their expectations for their daughters drop in junior high/middle school. Regardless of grades received in math courses, many parents feel that boys, compared to girls, have higher ability in math. Furthermore, they place higher value on math as an area of study for boys (Eccles, 1989). These parental expectations mirror those of our society and result in a myriad of signals that discourage adolescent girls from continuing to study both math and science. Furthermore, they reinforce the aspects of being a scientist, mathematician, or engineer (e.g. working alone in a highly competitive atmosphere) that work to exclude women. That exclusion enhances the masculine image of science. Descriptions of the scientific enterprise by Watson (1981) and Keller (1986) show the marginalization of women in the scientific workplace. My female doctoral students report subtle differences in access to professors, in mentoring, and in availability of resources that suggest that during graduate school the academy holds lower expectations for the future success of women scientists. Those expectations are solidly based on current conditions that result in lower pay, fewer rewards, and higher rates of under- and unemployment. Under the headline, *Women Join the Ranks of Science but Remain Invisible at the Top*, the *New York Times* described some of these socio-cultural inequities (Angier, 1991). The article was instigated by the recent announcement of newly elected members of the National Academy of Sciences, of whom only six out of sixty were women. The scientists interviewed identified the subtle exclusion of women from the collaborative groups. As Dr. Elaine Fuchs, professor of molecular and cell biology at the University of Chicago, said:

> There is a lot of friendship and social interaction that goes on at the scientific level, and a lot of important information is exchanged in those interactions. You may be at a complete disadvantage in your ability to do good science if you can't get inside information. And though in an ideal world it would be other- wise, men usually feel more chummy with other men (Angier, 1991, p. B6).

In addition, the scientists identified other factors; for example, the exclusion of many women from the conference circuit that is a critical link to scientific prominence, and the reluctance of many women to engage aggressively in the intense competitive debate typical of scientific discussions.

There is evidence that the entrance and persistence of women into technical fields will both demasculize and demystify science, math, and engineering. The example is medicine, where women have increased their proportion from 10% in 1970 to 18% today. Partly as a result, archaic internship programs have been phased out, a change that has also resulted in higher quality patient care. In addition, the practice of medicine is being affected as medical professionals become more responsive to a fuller spectrum of patient needs. Although it can be argued that many women are marginalized in the medical profession, their sheer numbers are changing both the training for and the practice of medicine.

My experience suggests that in order for similar changes to occur in science, we must begin by demystifying science in elementary and secondary classrooms. First of all, boys need to realize that girls can do science, and the best way for that realization to occur is to insist that boys and girls participate in science equally and together. Girls in single-sex schools need to be challenged to compete with boys in other contexts. Campbell's recent analysis of Asian and Caucasian boys and girls who were Westinghouse Talent Search Award winners reinforces that need. He found that even among this very selective and high achieving group, Caucasian girls scored significantly lower than all other groups on a mix of variables called *Technical Orientation*. Those variables included: high SAT-M scores, high scores on math and science self-conception, high scores on degree of persistence to a scientific college major, and moderate assessments of the amount of effort needed to excel in science and math (Campbell, 1991). According to both Eccles and Campbell, variables such as those identified as *Technical Orientation* influence achievement in science and mathematics. With their low scores on *Technical Orientation*, it is not surprising that, compared with Caucasian males and Asian-American females and males, significantly more Caucasian females choose non-technical college majors. In addition, other studies show that Caucasian girls avoid mathematics and science competitions and that their parents support that decision. This overprotective behavior was not found among Asian-American parents (Campbell & Connolly, 1987).

In Conclusion

Intervention programs in math and science have the same goal; improved achievement and retention levels for girls. However, in my opinion, success in science has been hampered by an over-emphasis on careers, attitudes, and interest. Intervention projects and programs must emphasize the experiences, activities, and skills that more boys, compared to girls, demonstrate. Furthermore, although girls need to be nurtured in science and mathematics by the increased use of discussions, laboratories, and small learning groups, eventually they must be introduced to the reality of doing science. Today that reality includes demonstrating that they are as competent as their male peers, particularly in competitive situations. ❖

FIGURE 1

COMMONALITIES AMONG TEACHERS WHO RETAIN GIRLS IN SCIENCE

- Kept classrooms that were attractive, well-equipped, and maintained
- Used non-sexist teacher-developed instructional materials to supplement text
- Were sensitive to sexism in science: used women role models and non-sexist career information
- Presented "girl-friendly science"
- Used laboratories, discussions, and tests more frequently
- Were respected, recognized, and supported within their community
- Participated in and encouraged students to enjoy science beyond school doors
- Emphasized careers and further education
- Used instructional techniques that encouraged creativity, further education, and basic skill development
- Had positive attitudes and encouraged students

SOURCE: JANE BUTLER KAHLE, "GIRLS IN SCHOOL/ WOMEN IN SCIENCE", 1983

Bibliography

American Association of University Women (AAUW), (1990). *Shortchanging Girls, Shortchanging America.* Washington, D.C.: The Analysis Group Inc.

Angier, N. (1991, May 21). Women join the ranks of science but remain invisible at the top. *The New York Times*, Science, B5, B7.

Ben-Chiam, D., Lappan, G., & Hourang, R.T. (1988). The effect of instruction on spatial visualization skills of middle school girls and boys. *American Education Research Journal*, 25, 51-71.

Campbell, J.R. (1991). The roots of gender inequity in technical areas. *Journal of Research in Science Teaching*, 28 (3), 251-264.

Campbell, J.R., and Connolly, C. (1987). Deciphering the effects of socialization. *Journal of Educational Equity and Leadership*, 7(3), 208-222.

Dossey, J.A., et al. (1988). *The mathematics report card. Are we measuring up?* (Trends and achievement based on the 1986 National Assessment Report No. 17-M-01). Princeton, NJ: Educational Testing Service. In Linn, M.C., & Hyde, J.S. (1989). Gender, mathematics, and science. *Educational Researcher*, 18 (8), 17-19, 22-27.

Eagly, A.H. (1987). *Sex differences in social behavior: A social-role interpretation.* Hillsdale, NJ: Lawrence Erlbaum Associates, Publishers.

Eccles, J.S. (1989). Bringing young women to math and science. In M. Crawford & M. Gentry (Eds.), *Gender and Thought: Psychological Perspectives.* New York: Springer-Verlag.

Jorde, D. & Lea, A. (1987). The Primary Science Project in Norway. In Kahle, J.B., Daniels, J.Z., and Harding, J. (Eds.), *Proceedings of Fourth GASAT Conference* (pp. 66-72). West Lafayette, IN: Purdue University.

Kahle, J.B. & Danzl-Tauer, L. (1991). Toward greater participation by women in science. In S.K. Majumdar, L.M. Rosenfeld, P.A. Rubba, E.W. Miller, & R.F. Schmalz (Eds.), *Science Education in the United States: Issues, Crisis, and Priorities.* Easton, PA: Pennsylvania Academy of Science.

Kahle, J.B. & Lakes, M.K. (1983). The myth of equality in science classrooms. *Journal of Research in Science Teaching,* 20, 131-140.

Kahle, J.B. (1985). Retention of girls in science: Case studies of secondary teachers. In Kahle, J.B. (Ed.), *Women in science: A report from the field.* Philadelphia, PA: The Falmer Press.

Kahle, J.B. (1987). SCORES: A project for change? *International Journal of Science Education,* 9(3), 325-333.

Kahle, J.B. (1990). The effect of teacher inservice programs on elementary students' achievement & attitudes in science. (Interim report). Oxford, OH: Miami University.

Kahle, J.B., Anderson, A.Y., & Damnjanovic, A. (1991, July). A comparison of gender-based attitudes in science held by primary students and teachers in Australia and the United States. Paper presented at the Australian Science Education Research Association, Gold Coast, Queensland.

Keller, E.F. (1986). How gender matters: Or, why it's so hard for us to count past two. In J. Harding (Ed.), *Perspectives on gender and science* (pp. 168-183). London: The Falmer Press.

Kelly, A. (Ed.) (1981). *The Missing Half: Girls and Science Education.* Manchester: Manchester University Press.

Licht, B.G., Stader, S.R. & Swenson, C.C. (1989). Children's achievement-related beliefs: Effects of academic area, sex, and achievement level. *Journal of Educational Research,* 82(5), 253-259.

Linn, M.C. (1990). Gender, Mathematics, & Science: Trends and recommendations. Paper presented at the Summer Institute for the Council of Chief State School Officers, Mystic, CT.

Linn, M.C., & Hyde, J.S. (1989). Gender, Mathematics, and Science. *Educational Researcher,* 18(8), 17-19, 22-27.

Linn, M.C., & Peterson, A.C. (1985). Facts and assumptions about the nature of sex differences. In S.S. Klein (Ed.), *Handbook for Achieving Sex Equity Through Education* (pp. 53-77). Baltimore, MD: Johns Hopkins University Press.

Linn, M.C., De Benedictis, T., Delucchi, K., Harris, A., & Stage, E. (1987). Gender differences in national assessment of education progress

science items: What does "I don't know" really mean? *Journal of Research in Science Training*, 14(3), 267-278.

Mullis, I.V.S. & Jenkins, L.B. (1988). *The Science Report Card: Elements of Risk & Recovery*, Princeton, NJ: Educational Testing Service.

Parker, L.H. (1985, Dec.). Non-sexist science education: An issue of *primary* concern. Paper presented at the Science Teachers Association of Victoria Conference, Monash University, Melbourne, Australia.

Prose, F. (1990, January 7). Carol Gilligan studies girls growing up confident at 11, confused at 16. *The New York Times Magazine*, pp. 21-25, 37, 45, 46.

Rennie, L.J., Parker, L.H., & Hutchinson, P.E. (1985). The effect of inservice training on teacher attitudes and primary school science classroom climates. Research Report No. 12. Perth: University of Western Australia.

Sells, L. (1976). The mathematics filter and the education of women and minorities. In Fox, L., Brady, L., and Tobin, K. (Eds.), *Women and the Mathematics Mystique* (pp. 66-75). Baltimore, MD: The Johns Hopkins University Press.

Tartre, L.A. (1990). Spatial skills, gender, and mathematics. In Fennema, E. and Lede, G.C. (Eds.), *Mathematics and Gender* (pp. 27-59). New York, NY: Teachers College Press.

Tobin, J. (1987). Gender differences in science: They don't happen here! In B. Fraser and G.J. Giddings (Eds.), *Gender issues in science education*, (Monograph No. 8) (pp. 37-45). Perth: Curtin University of Technology, Faculty of Education.

Tobin, K., & Gallagher, J. (1987). The role of target students in the science classroom. *Journal of Research in Science Teaching*, 24, 61-75.

Tobin, K., Kahle, J.B., & Fraser, B.J. (Eds.) (1990). *Windows into science classrooms: Problems associated with higher-level cognitive learning*. London: The Falmer Press.

Watson, J.D. (1981). *The double helix: A personal account of the discovery of the structure of DNA*. London: Weidenfeld and Nicholson.

Whyte, J. (1986). *Girls into science and technology*. London: Routledge & Kegan Paul.

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She was the first Chair of the now-defunct National Advisory Council on Women's Educational Programs (1975-77) and served as a member of the Presidentially-appointed Council until 1982. She also served as a member of an Advisory Committee on the Economic Role of Women to the President's Council of Economic Advisors in the early 1970s. In 1982 *Washingtonian Magazine* named her as one of Washington's most powerful women, and in 1988 the *Ladies Home Journal* repeated that citation, naming her one of the nation's 100 most powerful women. ❖

The Chilly Climate

~ *And how it affects girls in science* ~

Bernice Resnick Sandler

EVERYONE IS CONCERNED ABOUT getting more girls and women interested in science. And well they might be, for if we do not succeed, there simply won't be enough scientists in the twenty-first century. I want to talk today about a few of the reasons why girls and young women are less likely to be interested in science than their brothers, and what it is that teachers can do to encourage girls and young women to enjoy science and to think of science as a possible career option.

Now why is it that females are not as interested in science as their brothers? Although there are many dynamics that affect why girls are less likely to be interested in science, I want to talk about two major factors. One is what happens to girls in the classroom, and two, the school itself as a whole and the messages it gives to girls.

First let me set the stage by starting with stereotypes and different expectations for each gender, and how these affect what happens in school. After all, we know that boys and girls both do well in science and math in the early years, but as time goes on the girls begin to show less interest and, as a group, begin to fall behind.

Stereotyping

From the very beginning, boys and girls are treated differently, even in the cradle. We color code babies so we will know how to treat them. We let boy babies cry longer before we pick them up, and we talk to girl babies more often. Then when the girls turn out to be more verbal we say that this is inborn and that it is in the genes.

At an early age girls learn that they are not supposed to be as active as boys. Picture books and TV shows typically portray boys as active problem-solvers, while too many still show girls, if they show girls at all, as watching or being rescued by boys. Girls often have little knowledge of science — walk down the pink and purple part of a toy store. The science toys are not there, but are in the "boys'" part of the store.

By the time boys and girls first enter school they have received thousands and thousands of messages from the media, from their peers, from society, and sometimes even from their parents, that girls and boys are very different, that boys are expected to be more dominant, that boys are smarter, that boys are better in math and science. To the extent that we expect boys to do better in math and science, we may well be setting up self-fulfilling prophecies.

The Chilly Climate

But much more is going on than just stereotyping. I want to talk about the chilly climate—a series of small behaviors, often unnoticed by either the party who commits them or the person the behaviors are aimed at—behaviors that communicate to some or all females that they are not as worthwhile and are not expected to participate in the classroom or elsewhere as much as males are expected to participate.

I want to tell you how I got started on the chilly climate issues. A few years ago I was watching a morning television show and a researcher told how in mixed groups of men and women, women were much more likely to be interrupted than the men. I thought this was rather interesting information, although I had never noticed it before. A few months later, I was asked to attend a seminar for executives at the Aspen Institute in Colorado. There were fifteen business men (and I do mean men) and four women, all non-profit types like myself. The Institute was very proud of itself; this was the most women they had ever had. After a few days, I began to realize that the women were getting interrupted quite a bit. I didn't want to be paranoid, so I first checked with the other women to see if they had noticed the interruptions. The three women all responded in the same way: "Well, yes, now that you mention it." None of them had noticed the interruptions until I mentioned it to them. Because I'm a former researcher, and I wanted to be absolutely sure, I made a little chart showing the number of male interrupters, female interrupters, male interuptees and female interuptees. The next morning I didn't participate but merely observed and filled in my little chart. It turned out that the remaining three women had about double the number of interruptions of the remaining fifteen men. There was also a difference in the kind of interruption for men and women. For men, the interruption was really a continuation or development of their comments such as "What you are saying is that Confucius and Marx were not very far apart." For women, the interruptions were of a very different nature. For example, one woman's husband—a reporter for the *New York Times*—was attending the conference as a spouse. One of the interruptions consisted of "Well, what do you think Martin would say about that?"—an interruption that subtly or not so subtly communicated to her and everyone else that what she said was not quite as worthwhile as what other people (the men) had said.

I showed my chart to the two co-leaders after the class, trying to convey

the information as pleasantly as possible, without a confrontation, and with a smile. The men denied the accuracy of my chart (surely I had misunderstood, surely I had miscounted). I left the meeting quite dejected. But the next morning there were no interruptions of any kind for the women. I said to myself (and I'm not making this up) "Wow, this behavior is changeable, and I ought to get a grant to do something about it."

So under a grant from the Fund for the Improvement of Postsecondary Education (FIPSE) and some other federal agencies—these are your federal tax dollars at work—we began to examine our own extensive files, (we examined research done by linguists and communication researchers, including research done in classrooms at all levels), and to identify the ways in which male and female students are often treated differently. We gradually came to realize that many of the so-called "problems" of girls and women may indeed be related in part to how they are treated in the classroom.

- Girls and women do not participate in classrooms as much as males.
- Most young women major in the traditional "female" fields rather than fields such as mathematics and the sciences.
- For many females there is a drop in their self-esteem during early adolescence and again during their college years.

Initially we thought of these behaviors as happening only to girls, both majority and minority, but not to boys. Over time, we have gradually come to realize that minority boys may also be treated this way. Additionally, low status persons, such as working class students, disabled students, or students thought not to be as bright, may also be treated this way. So what I am talking about today not only applies to coeducational schools but to all-female schools as well. In the latter, a few females who are thought to be very bright may be treated very differently from the other females in the classroom.

The behaviors are not committed only by men. Often women faculty and other staff, even those most dedicated to equity, may inadvertently and unknowingly engage in the same behaviors.

We looked at two kinds of behavior: behaviors where females are ignored and behaviors where females are singled out and treated differently. These behaviors do not happen in every class nor do they happen all the time. But they happen enough so that they constitute a pattern—a pattern of behavior that dampens girls' ambitions, lessens their classroom participation, and attacks their self-esteem.

I am not going to talk very much about obvious overt discriminatory behaviors that still occur, such as the following:

- Negative comments about women, such as "women aren't as good as men."
- Comments that disparage females' intellectual ability, such as "Well, you girls probably found this boring," or "You girls wouldn't understand this."

- Comments, particularly from a male teacher, that divert discussion of a student's work toward a discussion of her physical attributes, such as cutting a student off mid-sentence to praise her attractiveness. While such comments may not seem harmful, and may even be made with the aim of complimenting the student, they often make females uncomfortable because essentially private matters related to the sex of the student take precedence over the exchange of ideas and information. Females are much more likely to be appraised for their attractiveness rather than for their abilities. For some females, the only praise they ever receive from teachers is about their appearance or the appearance of their work.
- Comments that rely on sexist humor as a classroom device to liven up the class. Humor can be used to lighten up a subject and relax people, but it can also be used to express anger and resentment. It can also be used to define the outsider or deviant group. Jokes demeaning to females are quite common, and sometimes jokes which ridicule women are told in classrooms.

Subtle Behaviors

The subtle behaviors, however, are much more interesting and actually more devastating. Because students are less likely to recognize that anything unusual has happened, the behavior has a subtle, damaging, but unnoticed, effect. Here are some examples of how some females may be treated in coeducational and single sex classrooms:

Females make more eye contact with males than with females. In single sex classrooms, the target students, those judged to be the brightest and those who grab for the attention, are most likely to be looked at. One female college teacher after reading our report discovered that when she asked a question, she looked only at her male students as if only men were expected to respond.

Teachers are more likely to nod and gesture in response to comments from males, and to lean forward and assume a position of attentiveness. When females talk, or when the females considered less bright talk, teachers may be inattentive, such as looking at the clock or shuffling papers.

In coeducational classes, teachers may group students in such a way so that it is clear that females are not considered as competent as the males. A laboratory teacher may insist that there be no all-female laboratory teams because the girls "can't handle the equipment on their own." Others may group the females together "so that they can help each other" or so "that they don't delay the boys."

There is a popular notion that in everyday situations females talk more than males. Yet in studies of formal groups concerning men and women:

- men talk more often than women
- men talk for longer periods and take more turns at speaking

- as mentioned earlier, men are more likely to interrupt women. You can see this in classrooms. A teacher asks a question and the boys call out the answer to which the teacher responds positively. When girls call out the answer, some teachers are likely to say, "In this class children, we raise our hands."
- men exert more control over the topic of conversation.

Not only do men talk more, but what men say often carries more weight. A suggestion made by a male person is more likely to be listened to, credited to him ("as Bill said"), developed in further discussion, and adopted by a group than when the same point is made be a female. No wonder that women and girls often complain of not being heard.

Teachers may also ignore some students, particularly females, even when they raise their hands. Boys are more likely to be called on directly as well as those female students thought to be among the brightest, while other girls are ignored.

Male students may be called by name more often. Sometimes faculty are surprised to find out that they know the names of proportionately more males than females. In all female classes, the target students are most likely to be called by name. Calling a student by name reinforces a student's sense of being recognized as an individual. Sometimes boys are addressed by their formal names—"John" and "James"—while girls are called by the diminutive version of their names—"Jody," "Suzy," and "Jeannie"—names which emphasize their "cuteness" and femininity and set them apart from the boys. Being called "sweety," "honey," or "dearie" by male teachers may do the same thing.

Sometimes coeducational classes are addressed as if there were no women present. "Suppose when you grow up that your wife..." For those of us who will never have a wife, the question cannot be answered; we are discounted as participants.

Sometimes male students as well as the female students assumed to be the brightest are coached for additional answers such as, "Tell me more about that," or "Why do you think that is?" while other girls' comments go unreinforced.

There is some research at the elementary level which indicates that teachers give brighter students more time to formulate a response. It is as if they assume that the brighter students are formulating the answer during the wait time, while the same pause by a seemingly less bright student is seen as "not knowing the answer." Giving some students less time to answer subtly communicates that those students are not expected to know the answer.

Another difference we noticed was that male students and those considered brighter students are more likely to be asked the harder and more thoughtful questions, such as "Why did the Revolution occur?" This is a question in response to which one can display one's cleverness, or, if one doesn't know the answer, one can try to fake it. Females are more likely to

be asked the more simple, lower order factual questions, such as "When did the Revolution occur?"

Perhaps more damaging is the tendency of some teachers to respond more extensively to males comments or to those few females considered exceptional. Thus a male's comment or that of a bright girl may elicit praise, additional questions and coaching, and additional intellectual dialogue with the teacher. In contrast, most girls are more likely to elicit the ubiquitous "uh-huh" when they speak—a comment that gives them little encouragement, little incentive to speak again, little reinforcement, and indeed, little feedback other than "You said something and I heard it."

Now why should these behaviors occur? Most teachers want to be fair and most believe that they are fair. Yet something is going on. There are several hypotheses that may explain these behaviors. Certainly the stereotyping with different expectations and perceptions about girls and females may be part of what is happening.

The Devaluation of Girls

Another explanation is the devaluation of what is female. Through our society, what females do is seen as less valuable than what men do. There have been numerous studies where two groups of people are asked to evaluate the same set of articles, or pictures of art, or resumes or whatever. Half of the names are female and half are male, but the names are switched for each group, so that the articles which have female names for one group are given to the second group with male names, and the articles which had male names for the first group are given to the second group with female names. What is quite consistent about these studies is that if people believe a woman was the creator they give it a lower rating than when they believe it was created by a male. Both men and women do this: they devalue those items ascribed to females. You can also see devaluation in how men's success is typically attributed to talent; women's success is attributed to luck or hard work.

The devaluation of what is female affects girls and boys at an early age. In another study, boys and girls were given the identical task to solve, and virtually all of the children solved the problem. The observers, who included both men and women, were asked, for each child, did the child solve the problem by luck or by skill. For the little boys, almost all of their success in solving the problem is attributed to skill, and for the little girls, almost all of their success is attributed to luck.

The relative lack of self-esteem that young and older women often suffer is in part related to the devaluation they experience in school, in their social lives, and in the workplace. This lack of self-esteem among females means that they are less likely to take risks, and more likely to blame themselves when things go wrong. Boys having a hard time with science are likely to say,

"This is a rough course," or "The teacher was no good," or "I was so busy with athletics that I didn't have time to study." Girls are more likely to say, "I'm not smart enough for science."

The devaluation that girls face is shown in many ways. Girls are less likely to get encouragement because they may be seen as less capable and therefore less likely to succeed. For example, a boy who is stuck may get encouragement to solve a problem on his own; we may be more likely to help or "rescue" a girl in the same situation, so that in a science class girls may get less hands-on experience when the teacher does the problem for them.

Here is another example of how devaluaton can have damaging consequences for boys and girls. Fourth grade boys and girls are asked to write how their lives would change if they were the other sex. The girls write about how they would be astronauts, and the kinds of adventures they would have. Many of the boys could not even answer the question. One writes that he would kill himself. To boys, being a girl is a terrible experience; it means that for boys, girls are worthless, not nice, not someone they can identify with. And what does that mean in terms of the relationships between boys and girls, and when they grow up, between men and women?

The Way Girls Talk

The devaluation may also affect our response to the way that girls talk. Females are far more likely to talk "politely", hesitantly, deferentially, and less assertively. They may use tag questions, such as "It's cold in here, isn't it?" Typically they may use qualifiers, such as "Well, maybe there is a chance that somehow there is a likelihood..." They also use what are called "hedges", such as "It's sort of hot in here," and they are more likely to apologize for what they say, such as "I'm probably not right, but..." or "This doesn't make much sense, but..." In contrast male speech is more likely to be definitive and decisive.

This more hesitant, uncertain, less assertive speech is often misinterpreted as being synonymous with a lack of knowledge or intelligence. Thus the style of the speech takes precedence over what has been said; the content becomes less important.

Should girls learn to speak like boys? Should women learn to speak like men? We all know what happens when women speak like men; they are called pushy, hostile, bitchy, and castrating. Ideally, both sexes need to be able to speak in both styles of speech and to choose when to use them. Both have their uses. Women's style of speech, with its openness and tentativeness helps other people join in the discussion while too assertive speech may turn people off. For example, some of us would choose not to enter into a discussion with an overly assertive person.

Let me give you another example of how "female speech" leads to different dynamics. If I am at a meeting and I say, "We need to have a

subcommittee to solve this problem," people have to respond in terms of agreeing or disagreeing with me. In contrast, if I say, "Well, what do you think? Would a subcommittee be helpful in dealing with this?" Now the discussion can be more truly collaborative; each can discuss the issues with the others. The focus is on the discussion and not on the relationship to me. What is important in all of this is to recognize that speech patterns are learned, that different speech patterns are useful in different contexts, and most important, that the style of speech is not by itself an indication of intellectual ability.

Before I go on to other aspects of the chilly climate, let me give you some suggestions for dealing with the subtle classroom interactions.

How to *Discourage* Girls from Participating in Class

- When addressing the class or asking a question, look primarily at males and/or the students you consider the smartest, as if they are the only ones you expect to respond.
- Respond to female comments with negative body language, such as avoiding eye contact, looking out the window or shuffling papers.
- Ignore male students when they interrupt female students, squeeze out females during a lab demonstration, or when they show hostility to females by commenting loudly or even hissing.
- Judge the content of what females say by how they say it. If a female speaks hesitantly or apologizes ("Maybe I'm wrong, but...") assume that she is not too bright.
- Respond to female's comments with an "uh-huh" or "okay," but offer precise, substantive responses when males speak, or when the females you think are brightest speak.
- Use examples that exclude females as participants ("When we were boys we did this..."). Using male-based experience as illustrations, such as sports or car repairs, makes girls feel like outsiders. (During the Gulf War, General Schwartzkopf talked about the "Hail Mary" strategy, and assumed that everyone would know what he was talking about. For those of us who are less familiar with football, his comments were a mystery; we did not understand and were turned off.)
- Joke about the women's movement, and if you are male, joke about your own male chauvinism as if this thereby excuses you from being a chauvinist.
- Make seemingly helpful remarks that disparage females' abilities such as "I know you girls need more help in math, but I'm going to give you some extra help."

Here are some positive things you can do:

Ways to *Encourage* Girls to Participate in Class

- On the first day tell students you expect them all to participate.
- Call all students by name, especially females.
- After asking a question, most of us wait less than two seconds before we call on the first person who raises a hand. Endure the silence. Count 1000, 2000, 3000, 4000, 5000, before you respond. Females often wait longer than males to put their hands up because they are thinking about what they will say. Males are more likely to raise their hands and think about what they will say as they say it.
- Watch for non-verbal cues, such as leaning forward or making eye-contact, indicating readiness to participate.
- When a student deserves it, offer praise ("I like what you said."). Because girls are more concerned about relationships and have less self-esteem, you get more response from your encouragement dollar when you encourage girls than when you encourage boys. Criticize, too, but in such a way that students want to work harder. ("I know you can do better. What you need to do is redo the experiment and keep an eye on the changes in the fluids.")
- Keep a teaching diary, especially at the beginning of a semester. Record in it which students are contributing and which ones are not. Make a point of encouraging the "silent" students.
- Avoid the so-called generic "he" or "mankind." When you say "he or she," you communicate your awareness of women's concerns. Teachers can be role models in the way they behave and talk about issues involving women. Using the so-called generic "he" to represent both men and women gives a message that women are outsiders; using "he or she" or "men and women" tells students that females are important and a part of life. One study gave children hypothetical book titles and asked them to draw book jackets for them. For titles such as "The History of Mankind," children drew pictures of men. For titles such as "The History of Humanity," they drew men and women, boys and girls.
- Mentally divide a class into quadrants and occasionally say, "I haven't heard from the back right for a while." This will encourage people in that quadrant to participate.

Now even if you do all of this, there are other aspects of the classroom that make it chilly, and that is the matter of teaching style. Girls are more likely to be concerned about relationships to other people, and generally boys are more competitive. Obviously, this is a generalization and not true all of the time or of all boys and all girls. Yet to the extent that girls are less competitive than boys and more interested and comfortable in a collaborative and cooperative situation, presenting science as a highly competitive endeavor and using a highly structured and competitive teaching style can turn girls off.

What can we do to counteract the discomfort with competition experienced by many girls? How can we build on the strengths they have in developing collaborative relationships?

A student-centered pedagogy is most appropriate, and, of course, it is also useful for boys as well. The aim is to foster the student's sense of competence and independence, as well as foster the ability to work well, both independently and with others. The following may be helpful:

Classroom Strategies

To foster the student's sense of competence and independence, as well as foster the ability to work well both independently and with others:

- **Cooperative Learning—especially by using small groups and independent Projects**
- **Student-selected topics for projects**
- **Student-selected activities for independent and group projects**
- **Class discussions, in contrast to lecture-oriented classes**
- **Experiental learning, such as field trips and hands-on experiences**
- **Student evaluations of each other's work**
- **Ask students at the end of each class to anonymously write down the main part of what they learned and what they do not understand.**

~ Adapted from Bernice R. Sandler, Math and Science for Girls, June 17, 1991

But even if we add cooperative and collaborative learning to our earlier classroom interaction strategies, there are still more factors that make for a chilly climate. The first of these refers to coeducational and coordinate schools, and the second refers to all schools.

"Boys will be Boys"

First is the relationship between boys and girls. Clearly members of each gender need to identify with their own sex and to be comfortable with this identity. Unfortunately, our society often encourages boys to develop their sense of gender identity by using females as what sociologists call "a negative reference group." Boys know that they are "real" boys because they compare themselves to the lesser group, girls. After all, what is the worst thing you can call a boy? A "sissy," in other words, a girl. By teasing girls, a boy begins to feel good about himself—he is "better" than girls, and teasing makes him feel more like a "real boy". Moreover, by putting down girls and females, he can get closer to his buddies. For many males, harassment of females is a way to show other males how manly they really are. Sometimes it is part of the bully syndrome; the boys feel better by picking on someone they perceive as weaker. Harassment is a milder form of assault. At its extreme, it sets the stage for rape. But even if it does not do that, it poisons the relationships between males and females because power becomes the major currency of the relationship, rather than intimacy, friendship and sharing.

Although gender identity and "bonding" are important for growth and development, they do not excuse discrimination. It is not "cute" nor is it necessary for boys to be nasty to girls. When gender identity and bonding become hurtful and discriminatory, it is time to intervene. The hurtfulness of harassment increases girls' lack of self-esteem. It validates the devaluation that they experience from the media and from society.

One rough way to mentally test out whether some behaviors are discriminatory is to ask if the behaviors would be allowed if they involved different races rather than gender. Let me give you an example of what I mean, although the example is from a college campus. A fraternity and sorority sponsor an "Ugly Woman Contest" in which the fraternity members, with the assistance of sorority members, dress up as women. The contest is approved by the school's Greek advisor in the office of student affairs. However, several of the men dress up as black females. Now a protest ensues, and both the fraternity and sorority are disciplined. We do not tolerate students demeaning blacks or other groups, but we tolerate females being demeaned, and we call it "boys will be boys".

We recently did a report about peer harassment at the college level and were stunned to find out that some 70 to 90 percent of female students have experienced some form of harassment from male students to which they reacted very negatively. There has been very little written about harassment

in secondary schools but there is every indication that peer harassment is increasingly a serious problem.

What can we do about the climate between boys and girls? At the least, have a policy which clearly articulates that the school will not tolerate harassing behavior. Enforce the policy. Develop an educational program about the relationships between males and females. While it is beyond the scope of this talk to describe what kinds of educational programs would be appropriate, let me focus for a few moments on some things teachers can do to minimize conflict and competition between the sexes.

Toward Equality

Children can be grouped in many ways. Grouping them by sex accentuates the difference and often gives off negative messages. Games where it is the boys against the girls do not help boys and girls to get along with each other.

In some games and tasks, the "captain" picks out the team. Most children will pick those of their own sex first. Here is an opportunity for discussion. Ask the class what's going on. Preadolescent children are wonderfully moral at times, and they can be very responsive to discussions about prejudice and fairness. You can also choose captains who are both of the same sex (two boys or two girls) so that the tendency to pick one's own sex is somewhat negated.

Most important, however, is for teachers to intervene and condemn bad behavior when it occurs. Allowing it to continue without doing anything gives the message that the behavior is okay, that it is all right to demean females and that no one cares.

But even if we do all of these things, there is still one more major aspect of the chilly climate, an issue of concern to all schools—coeducational or single sex.

The Curriculum

The school, through its curriculum, gives messages to its students about the lives of men and women. All too often, our schools echo the messages of our culture. Yet what schools should ideally do is not only build on the strengths that students bring to them, but also help free individuals from the constraints of their experiences. In a very real sense, the school should act as a counterbalance to what girls are learning elsewhere about women in the world and about women in science.

The school needs to act as a counterbalance to those trends in our society that hurt the development of girls and women. Schools must set out deliberately to provide a different kind of experience from what young women and girls would find elsewhere. You can call it re-training, de-conditioning, nurturing development, or whatever else you choose.

Perhaps what girls need most is a keen sense of what women's lives are like, what the barriers are, and the knowledge that overcoming these barriers is indeed possible. To develop a positive self-image, and to increase self-esteem, we need to provide girls with the knowledge of female experience. Just as black studies have helped blacks develop a sense of pride in their past and in themselves, women's studies provide a similar experience for girls.

If we do nothing in our curriculum, we reinforce the stereotypes that women are passive and less important than men, that women are there for men's benefit, that women are not the achievers in this society and that men are, and that women are not scientists. If we do nothing, this is what we are teaching, whether we teach history, or English, or even science. The content of the curriculum is a powerful tool, and to include the study of women is perhaps one of the most wonderful ways to increase the self-esteem, ambitions, and participation of girls. At the college level there are over thirty thousand courses in women's studies, close to 600 programs whereby students can major or minor in women's studies, and over 150 programs where faculty incorporate the new scholarship on women into the basic curriculum.

How can we do this? At the very least, a teacher can point out sexism when it occurs in the texts that we use, in our language, and in classroom dynamics. Teachers can look at the language, pictures, and content of textbooks. Instead of preaching about sexism, a teacher can ask probing questions to help youngsters identify the problem and insure that the insights they develop will be their own. Questions such as "What's wrong with this picture?" or "What's left out?" can begin the conversation. You can also be light about these issues. You want to communicate that it is a problem, but one that can be corrected.

The major way in which schools can do this is to address women's issues in the curriculum and incorporate the new scholarship on women into the regular curriculum, whether it is social studies, literature, or science. I want to close with an example of how the curriculum can help girls and women. We are all familiar with the story of Adam and Eve. What does that story teach us about women: that they are evil and should listen to their husbands. Let me give you a feminist interpretation which comes from Phyllis Trible. Eve is a mover and a shaker, and Adam is not. Eve contemplates her decision, she discusses its theological implications with the serpent. She thus is making a considered decision to taste the apple. Adam does not make a considered decision. Eve merely gives him the apple to eat and he does so, without question or comment. Both have eaten of the apple, but their behavior is very different. Phyllis Trible writes:

> "...the woman [Eve] is more appealing than her husband. Throughout the myth she is the more intelligent one and the one with

> greater sensibilities.... The initiative and the decision are hers alone. She seeks neither her husband's advice nor his permission. She acts independently. By contrast, the man is a silent and bland recipient. 'She also gave some to her husband and he ate.' His one act is belly-oriented, and it is an act of quiescence, not of initiative. The man is not dominant; he is not aggressive; he is not a decision-maker.... He follows his wife without...comment, thereby denying his own individuality. If the woman be intelligent, sensitive and ingenious, the man is passive, brutish and inept."
>
> ~ Trible, Phyllis "Depatriarchalizing in Biblical Interpretation" in *The Jewish Woman*, edited by Elizabeth Koltan, Schocken Books, N.Y., 1976.

Eve is truly capable of making change; Adam is merely a follower of others. By looking at this story with different eyes, we see a very different Eve than the one we thought we knew. And now what do we learn about women? Which of the two interpretations will help young women become independent and strong and questioning? Which will more likely help a young woman to be interested in science?

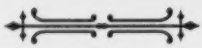
It is only when girls and women begin to examine their lives as females in our society and throughout history that they can build new sources of strength within themselves, and to cast off old stereotypes which constrain their lives in the same way that bound feet constrained the lives of Chinese women.

Let me end with a new "Biblical" revelation, said to have been discovered in the Middle East, by a woman archeologist of course. It is characteristic of the new mood of women, and you will probably recognize the paraphrase:

> And they shall beat their pots and pans into printing presses
> And weave their cloth into protest banners
> Nations of women shall lift up their voices with nations of other women
> Neither shall they accept discrimination any more.

Now that may sound apocryphal, but I suspect it may yet prove to come from the Book of Prophets, for what women are learning are the politics of change, and the politics of power. And our schools, and the nation, and the world will never again be the same. ❖

Selected Materials



The Cabbage Pack, Maths in Work, Mary Harris, Project Leader. Department of Mathematics, University of London Institute of Education, 28 Woburn Square, London WC 1H 0AA.

Common Threads, Catalogue of Exhibition of Mathematics and Textiles, Maths in Work, Department of Mathematics, University of London Institute of Education, 28 Woburn Square, London WC 1H 0AA.

EQUALS Handbook Order Form, Laurence Hall of Science, University of California (Berkeley).

"An Example of Traditional Women's Work as a Mathematics Resource", by Mary Harris, *For the Learning of Mathematics*, November, 1987.

"Girls and Mathematics: Some Lessons for the Classroom", Economic and Social Research Council, April, 1988.

A Seminar on Women and Mathematics: an Expanded Syllabus, by Miriam P. Cooney CSC, St. Mary's College, Notre Dame, IN., 1989.

Timeline, Women and Mathematics.

"Why Males Outscore Females on the PSAT/NMSQT: Big Surprises and New Hopes", by Roger McCaig, *College Prep* #7, The College Board.

"Women and Mathematics: Is There an Intersection?", by Leone Burton, *IOWME Newsletter*, April, 1987.

Women in Science and Mathematics Bibliography, compiled by Phyllis Zweig Chinn, Humboldt State University, Arcata, CA 95521.

Women in Technology Poster (Alcoa).

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Dr. Jacobs has written extensively in the area of mathematics and computer equity. She has edited *Perspectives on Women and Math*. She is also the founder and first president of Women in Mathematics Education, an international organization.

The following section is a slightly edited transcription from a tape of Dr. Jacobs' presentation. ❖

Women's Learning Styles

~ *And the teaching of mathematics* ~

Judith Jacobs

WHEN I DID MY DISSERTATION in 1973 mathematics was interpreted as a male domain. It still is a white male domain—mathematics is something that males do. I have a whole new interpretation of that about which you will hear later. Important recent data from the NAEP (National Assessment of Educational Progress) shows that girls in high school perceive mathematics as something that it is okay for them to do. But 17-year-old boys think that mathematics is not something that girls should do, and who has greater influence on high school girls than 17-year-old boys? So the point made by Jane Kahle last night is one I need to make again, and that is, any intervention you are going to do has to address males as well as females.

Mathematics Perceived as a Male Domain

Bernice Sandler talked about teachers' differential treatment of males and females, so I will not go over that again except to admit that I am as guilty as anybody else. In an elementary methods class that I taught, there were 25 students, 23 women and 2 men. I found myself spending more time talking with the men before class started than talking with the women. Now, whether it is that they demand more time or it is my natural inclination to give it to them, I don't know. All I know is that it is something I need to monitor or else I am as sexist as those who discredit females intentionally.

The next is the perception of self as learner of mathematics, which has two aspects, confidence and liking to do mathematics. How confident do you feel about learning mathematics? This is what the AAUW study examined. Girls like mathematics; they don't feel that they are good at doing mathematics. And, again, from elementary to high school we get a large drop in the number of girls who perceive themselves as good at mathematics. We need to do something about the confidence, because that will determine whether or not they decide to pursue mathematics.

The Causal Attribution Theory

The next area is causal attribution theory, which you have already heard about twice. Let me discuss this from another direction. Causal attribution theory looks at two factors; the locus of control, and stability. You have internal or external control, and you have stable or unstable. The effort one is able to put to a task is unstable; ability is stable. Task difficulty is external; luck and help available are external. If a teacher gives an exam and the students don't do well on the exam, girls and boys respond in different ways. Girls tend to think that they did not do well because they are "dumb". Boys think that the test was unfair or the teacher was unfair, but that it had nothing to do with them. As teachers, we know what to do in these situations because we all learned how to take students with negative self-image and say, "No, no, that's not true." Let's look at this another way. You give an exam and a student gets a 100% plus extra credit; it is an absolutely superb paper. You hand it back and compliment the young woman and she responds, "You're the best teacher of math I've every had. I couldn't have done it without you." We all like hearing that, but as teachers you have to say, "No, you did it." We tend, as women, to use that external reason for our success. We do not take credit for our accomplishments. And so even with that student who is getting an A in your class, you have to be careful about how they think about their success. You need to let them know that it is internal, and it is stable, and that it is going to happen again, and that they don't need you to do well in mathematics.

Autonomous Learning Behaviors

Some recent studies in mathematics education have come up with something called autonomous learning behaviors. A graduate student at the University of Wisconsin at Madison, Mary Anne Koehler, did a study in which she evaluated students' achievement in mathematics. She was really looking at the teacher of the classroom rather than the individual students. She studied two ninth grade algebra classes. In one classroom, the teacher was very supportive and did a lot of cooperative learning activities in a non-threatening atmosphere. In the other classroom, the teacher was extremely demanding, told the students they had to do the work on their own. The results showed that the students, both males and females, in the autocratic classroom did better than the students in the cooperative, collaborative classroom. So the conclusion is if you have autonomous learning behaviors, choose to engage in high level mathematical tests, prefer to work independently, persist with the task when the task is perceived as difficult, you will do better in mathematics. These autonomous learning behaviors come from four characteristics. Now look back to the other reasons: perceived usefulness of mathematics, confidence and ability to do mathematics, sex

role congruency with respect to doing mathematics (that is, mathematics as a male domain), and causal attribution theory. These are the behaviors that give us autonomous learning behaviors.

Here is a study that shows that a cooperative environment does not work. What do you do with that information? To me, for the last couple of years, as I have been reading the research, I keep hearing the same kinds of things. I start hearing another theme, and it says that the way to get women to achieve in mathematics and science is to get them to be like men. I have a little trouble with that.

Three recent issues in the news provide insight into this perspective. First, the **mommy track**, the very fact that women who are career women should be provided different opportunities and responsibilities than those women who wish to divide a career and family. Second; the **daughter track**, this was a cover story on *Newsweek* magazine, the average woman spends seventeen years raising children and eighteen years helping aged parents. The third is **parental guilt**, which shows that eighty percent of mothers, but only twenty percent of fathers, would like to quit their jobs and spend more time with their children.

I think that a lot of what we have been doing is dealing with "Why can't a woman (who is going to be successful) be more like a man?" I think this is the wrong question. We need to look at these things differently. Most of you, I am sure, are familiar with Carol Gilligan's work, *In a Different Voice: Psychological Theory and Women's Development*, where she found that women are different from men, not better and not worse. Different. That concept of different being of equal value is a very hard one to deal with. Women want relationships and men are more concerned about being fair. But the symbol for justice is a blindfolded woman who carries the scales far away from her body, which says to me that what they are valuing is not what I value. We need to make a fundamental decision as to how we are going to handle this. You heard both last night and this morning that you need to help girls be like boys, that they must become more competitive—both speakers said that, and I want to take a very different view.

I think the reason that the girls did not do as well in the cooperative learning environment as they did in the autonomous learning environment is that we really have not studied what good cooperative, collaborative learning is. I contend that the way women learn, the way women act and interact is different from men—not better or worse—but different, and that we need to address those differences rather than to suggest changing who and what we feel comfortable being.

Applying *Women's Ways of Knowing* to our Work

I want to talk about Belenky, Clinchy, Goldberger, and Tarule's work *Women's Ways of Knowing*, and its implications for learning mathematics. If you have not read this book, it is something that you need to read. It is available in paperback. There is also a book called *Making Connections* (edited by Carol Gilligan, Nona Lyons, and Trudy Hammer) that you should also read. *Women's Ways of Knowing* did this: it studied women from various social classes, ethnic groups, educational backgrounds—this was not a study done on college seniors—and looked at how they learned. The authors analyzed how women learn and know things. This is different from Perry's analysis which was done on Harvard undergraduates when Harvard was an all-male school and therefore did not really reflect how women think. I am going to put this analysis in the context of the concept "the base angles of an isosceles triangle are equal," taking a look at what a student would say if she were dealing with that theorem.

Silent Knowing

The first level of knowing something is silence. You accept the authority's verdict as to what is true, and at this level an inner voice is telling you that teachers think that the base angles of an isosceles triangle are equal. Notice that the individual makes no claim that he or she owns this knowledge in any way. This is an awareness that something is going on.

Received Knowing

The next stage I am going to talk about is that of received knowing. These stages are not as rigid as Piaget's, in the sense that you must be pre-operational before you can be concrete operational, but they are a hierarchy of stages. You learn by listening to the voice of authority; the speaker is not the source of knowledge: "I know that base angles are equal because my teacher tells me so." I think a lot of my students know mathematics that way. When I taught high school mathematics, I used a device that used to be called the "postman's story"; now we call it the "letter carrier's story," in teaching addition and subtraction of signed numbers. The story went: the letter carrier brings you a check for two dollars and a check for four dollars. Are you richer or poorer as a result of the visit? You are richer. How much? Six dollars; and you write on the board +2 plus +4 = +6. Okay. The letter carrier goes to the next house and brings a check for five dollars and a bill for seven dollars. Are you richer or poorer? You are poorer two dollars, and so on. We go through this story many times. One day we are getting into the lesson and I do the problem: -2 plus -5 is -7. Belinda White, who, on the first day of class came in and said, "I failed algebra last year. I have to sit in the front of the

room and really pay attention," said, "-2 plus -7 is +9." "Belinda, tell me the letter carrier story." "The letter carrier brought a bill for two dollars and a bill for seven dollars." "Are you richer or poorer?" "Poorer." "How much?" "Nine dollars." "Okay, Belinda, what is -2 plus -7?" "+9." "Try one more time." And then I did something that we don't do often enough. I asked her, "Why, Belinda?" Does anybody know what her answer was? That is right, "Two negatives make a positive." This is a received learner. She forgot the whole message. If we have students who are received learners, that is the kind of knowledge they have. So the fact that she said +9 when we talked about the fact that you owe nine dollars was totally irrelevant. The rule is the precedent. This is your received knowledge.

Subjective Knowing

The next stage is called subjective knowing. This is the inner voice that tells you: "I only know what I feel in my gut." There are two versions of that. The male version is "I have a right to my opinion" and the female version is "It's just my opinion." For example, I know that the base angles of an isosceles triangle are equal, just look at them, they are equal. One of the problems we have had in mathematics and science is that the people who have been doing the research on feminist pedagogy are basically humanities people, sometimes social science people, and so when you talk about such things as the beauty of a poem, you can deal with "It just really feels good" or "It sounds good to me." It is a very different thing in mathematics and science. One of the problems that we have is that not enough of us who are in mathematics and science have taught the women's studies course and looked at our own teaching and learning from that perspective. It is very hard to deal with this type of knower in mathematics because we all know how to come up with the example that shoots the theory; "Do all altitudes of a triangle intersect inside the triangle?" No, not if you have an obtuse triangle. But it does not feel right this way. This is subjective knowing.

Procedural Knowing

The next kind of knowing is called procedural knowing. There is another real difference between Piagetian stages and *Women's Ways of Knowing* stages. You do not automatically go from subjective knowing to procedural knowing with maturation. It requires some sort of instruction. It does not have to be from a teacher, but there has to be some intervention that lets you look at the validity of an argument. There are two types of this procedural knowing, separate knowing and connected knowing. Those of you in mathematics know that separate knowing is the "only way" to know mathematics. Separate knowing looks at propositional logic, you prove things. I know that the base angles of an isosceles triangle are equal, but

maybe not all base angles of all isosceles triangles are equal. I need proof. Connected knowing is where you look to the circumstances that led to the perception. *I* know that it looks that way, but what about the triangles that *you* looked at? Did it also hold for you? These are both procedural knowing; one is separate, one is connected. Here are the words that we associate with those two types of knowing. Which type is more valued in mathematics and science? Separate knowing. But guess what? Connected knowing is a more natural style for women.

Looking at these two lists causes real problems for me. I took traditional programs. I know how to prove things in mathematics. I can do things separately. But for me to really believe something, I need to be connected to it. It has to do with what we mean by proof. It also has to do with the difference between hard science and soft science. Just think of the words and how biological they are, and how they are gender tied. If I believe that women know things differently and have a style that is different from men, and if connected knowing is the way women are most comfortable, what does that mean for me in my mathematics teaching? It is at this point where I think we get down to my new meaning of white male domain, which is that mathematics and science are defined by the way men did them. I question whether it is inherently true that this is what mathematics and science is. Now, I know that this is a very different view of the world for a mathematician. I can know something even though I did not write a two-column proof.

In science we say, "Let's isolate all the variables so we can find the one cause of something..." Yet, what Barbara McClintock's research did was to look at the combination of factors that cause something to happen not separately, but in connection. Another aspect of the male way of viewing the world versus the female way is that men tend to see things in dualistic ways: right/wrong, nature/nurture. Think of the dualistic ways we deal with things and yet, if we look at it as connected, we can ask the question, "How does one's nature interact with one's environment to cause something to happen?" And if you took that same person and took them into another environment, would he or she be different? That is not nature versus nurture, it is connection. It is my belief that we need to start looking at mathematics and science with a respect for connections. And, in science, I think that when we ask a research question, we need to be prepared to live with the consequences of the answer and the effect it has on society.

When we decide what to look at in science, there should be a value component. Don't tell me that science is value free. It is not. I might accept that math is value free—I am not sure about that—but science certainly is not. I think we need to ask, "What is the consequence of this kind of research?" We all had an experience, and I am sure it was as mind-blowing to you as it was to me, to watch on our television screen the sight from an airplane as they carefully dropped a missile down between two buildings in Operation Desert Storm. I think you have to ask, "Is that where we should

put all our money? All our efforts in technology? Would it not be better to put all that time and energy into more productive science?" For example; artificial limbs. There are bionic limbs that use the muscle contractions in the part of an arm that is left after someone has had an arm amputated to operate an artificial hand so that a child born without one hand can learn to pick up a toy with her artificial hand. I think connected knowing should play a value-laden part in how we know things.

In mathematics, because mathematics has nothing to do with values—we all know it is abstract and totally irrelevant (that is what people tell me, anyway)—we need to deal with the concept of proof. There is a standard theorem in mathematics, "The sum of two odd numbers is even." At the first level students do this with twenty-seven examples. It always works, it has to be true. That is not a proof for me. It is an induction, but it is not a proof. I have worked with first graders, as well as junior high and high school and college students, on the same theorem, but what I use as my model are egg cartons. In an egg carton, the kind that comes in two lines, an odd number is one with a section sticking out. An even number just has pairs of sections. I don't care how many you attach to the back of these, if it is odd it has this little thing hanging out, and if it is even, it doesn't. And when you take these two egg cartons with these little things hanging out and put them together, what happens? They match up, and guess what? You have no little things hanging out. Now, how different is that explanation from $(2a + 1) + (2b + 1) = 2a + 2b + 2$? The plus ones go together and make a two. I sincerely believe that these proofs are equally valuable. The individual who can explain the egg cartons matching up knows what an odd number is, knows what addition is, and knows how to put them together, and what happens when you do.

Now, as I am a professor of mathematics, they think I'm wrong. And that is something we have to deal with, and I will talk about that in a little while. So I really think, even in mathematics, when you have separate knowing, there is a way of looking at the concept of proof that is not our standard way of doing proof. If what we really want from people is an understanding and ability to use mathematics, then I think we can do that in a different way.

Constructed Knowing

Let me finish the stages of knowledge. The last stage is constructed knowing. You have probably all heard about constructivism, having children construct their own knowledge. Constructed knowledge is where we want to go, because if you put ideas together yourself, you will really know it. If somebody else says, "Look, watch me do these egg cartons," it was not constructed by you and you don't know it as well. Constructed knowing is the last level. The important thing is that you can be a constructed knower without being a separate knower. I really do believe that if you are going to

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be a mathematician, you have to learn to do separate proofs, but how many people in this world are going to be mathematicians? Is it not more important to have a concept of proof that is broader than the formal proof we do in mathematics? As a citizen in society, I need to look at information which is not given with statements and reasons, and decide if the information being given to me is valid. Evidence of proof needs to be dealt with, and there are more ways of doing it than being axiomatic.

So, I present *Women's Ways of Knowing* as a vehicle for looking at how women can come to know mathematics and science in a way that is different from the way that white males have done it for centuries. We need to talk about strategies for teaching that build on that. One of the things that I do want to share with you is this idea of mathematics as a male domain, which I now have decided has to do with the way men do mathematics, as opposed to mathematics being something that men do, and I want to take a look at the language we use in the mathematics class. We say, "We are going to *tackle* those problems." I think we are going to interact with problems. We are going to integrate them into our consciousness so we know how to solve them. I know it may seem silly, but I think it matters. Just as when you hear "mankind" and females don't include themselves, I don't like tackling things. It is not one of my ways of doing things. The use of language in mathematics is something that we need to look at.

The view that mathematics is a male domain, and here is an area in which I disagree with our previous speakers, is something to be challenged. We have mathematics teams and we go to competitions. You have a team of five students? Do they function like a team? No. Those are five individuals from one place against five individuals from another place, and it is the individual scores that count. We are told that we should encourage girls to do that. Get into the mathematics competitions. What I want to do is to have mathematics teams and give each team a problem to solve. Let them work together to solve it and come up with the best solution rather than compete in the spirit, "I am really against you. I've got to be better than you even though you are on my team." I really think that the way we teach mathematics is a male way of doing things, and I do think that we can be creative in finding other ways of doing things which would make girls, young women, happier and more willing to be in that area. Why should a girl enter a mathematics competition if competing with people is not what she is about?

Addressing *Women's Ways of Knowing*

What are some of the techniques you can use to address *Women's Ways of Knowing*? The first is **cooperative learning**. You have heard about that. You are going to hear more about it this afternoon. Another thing to do in a classroom are **writing activities**. One approach is with **reaction papers**. When I am teaching a mathematics content course, somewhere during the

lesson I say, "Okay, stop and take one minute—that's all you have—write down what you have learned, write down what you have a question about." I collect those and use them for my next session to see what I missed and what I did not miss. Stopping in the middle often gives students the chance to say, "Wait a minute. I don't understand" and ask a question. Because if all you do is talk the way I'm talking at you here, by the time you get to the end, students forget what their questions are. So reaction papers can be used in mathematics classes. They let you say that probably the most important thing you could know is what you don't know. It is another way of saying that the stupid question is the one that wasn't asked. If *you* can identify what you don't know, I can help you much more easily than if *I* have to identify what you don't know. We need to help all students be more reflective.

Before continuing, I want to talk about the concept of generalizing. I am generalizing in this presentation, but in a mathematical sense. In mathematics, when we say "in general", we know that it means for every occurrence. That is not what we mean in social science. When I say that women respond a certain way, it means that it is *more likely* that women respond in a certain way. It does not mean that no man responds in that way, or that there are not some women who don't respond in that way. It is a generalization in a very different sense. Since you are in mathematics and science, I think the whole concept of generalization is something you need to look at because generalization has very different meaning in different content areas.

Journals, Papers, and Reports

Another writing technique from feminist pedagogy is the use of journals. I find that when my students write about how they feel as they are learning mathematics, as well as how they are doing in mathematics, they have amazing growth. Journals are a pain in the neck. You have five classes with twenty-five to thirty students in each class. When are you going to read these things? It takes a lot of time, but it is valuable. One of the things that I found with some of my students, and again, most of these people are women who want to be elementary school teachers of mathematics (or rather elementary school teachers who have to teach mathematics), is that they become aware of mathematics in the world in which they live. I have found that the students will come and start writing, "My goodness there's geometry all around us when I am teaching a geometry course," and they start seeing the world very differently. So, I think that a journal is another way of teaching mathematics and learning mathematics.

Another way to incorporate writing in our activities is to have people do papers, reports in mathematics. When I went to school—in fact, the reason I became a math major was that—you did not have to write term papers. Now I think students should write papers in mathematics and science. They should write about what they are interested in. It may be that you are going

to get a lot of papers on astrology. That is okay, because they need to do a lot of studying about math and science to understand astrology. We all know that next year is 1992, 500 years from 1492. Everybody is going to be doing a whole lot about Christopher Columbus. Well, that leads to opportunities for looking at longitude and latitude and how people used to navigate. That is a lot of mathematics. There are all sorts of areas. Writing papers is one way that they can express themselves and learn mathematics.

Students Watching Students

Another method in a mathematics class is to have students watch other students do mathematics. When you are in the process of doing a problem yourself, you are fully engrossed. You cannot think about how you did it. If I watch you do the problem, I can ask "Why did you do that?" The student learns to communicate about mathematics. Those of you who are familiar with the Curriculum and Evaluation Standards of the National Council of Teachers of Mathematics (NCTM Standards) know that communication is one of the primary standards. Communicating about mathematics is not necessarily answering the test question that the teacher wrote for you, it is the explaining of your answer. The kind of thing that I deal with is "What does .42 followed by a cent symbol mean? Teaching a class of future teachers, one young lady went home and spoke with her father, and her father said, ".42 with a cent symbol was the same as 42 cents," and I said, "No." She went home and she told her father that, and he was very upset. How do I deal with that? I said, "Let's forget about this one. Since this is the problem, let's not deal with that. Let's look at 50%; 50 with a percent symbol. What does that equal?" "One-half." There was no doubt about that one. She knew the answer. "Why does it equal one-half?" She didn't know, and I said, "Did you know that the percent symbol is really "multiplied by 1/100th" and if you had 50 times 1/100th, you would get 50/100ths and that equals one-half." Then we did some other percent problems. "So what would .5 percent be?" "It is .5 times 1/100th, which is 5/1000ths," and I used percents which she was more comfortable with to say, "Well, I've got news for you. It is the same thing with the cent symbol. That cent symbol means 1/100th." Talking about mathematics helps people learn. Because I was trained as a typical mathematician, I would much rather write "Sigma goes from 1 to N" for a sum using symbols, but many of our students don't function very well with symbols. What we need to do is observe how people do mathematics and discuss it. That is one mode that will help teachers introduce feminist pedagogy into the classroom.

There is another example that I use with respect to communicating mathematics. When you are teaching students about exponents we ask, "What is X^5?" The typical response is: "X multiplied by itself 5 times." But this is wrong, X multiplied by itself 5 times gives you X^6. If every time a

students sees X^5, a student can think, "X times X times X times X times X." I really don't care if what they say is "X multiplied by itself 5 times." But I can only let them do that when I know that they understand it. We have to be careful about language and the use of language, and yet I think you have to, at times, be incorrect to be correct.

Interviews and Presentations

Another technique for introducing feminist pedagogy in the classroom is through interviews. One of the things I do is to have a student interview another student on how he or she solves a problem. Then the interviewer has to write up the other person's solution. Again, this deals with communication about mathematics. It uses activities that we never thought were part of the mathematics class. Even science is ahead of us on that. In science, you always work in teams, mainly because there is not enough equipment. We became very technological and now we have microcomputers. Microcomputers are designed for one person to use. It is my contention that if you have a lab with ten computers, you should have a class size of twenty, because two on a computer is really the best way of doing it. Most of my male colleagues say, no, the class size is ten because they should not be talking to each other, they should be doing all the work themselves and always have hands-on. I really believe that committee work in mathematics is as valuable as lab partners in science. In social studies you do committee work, in language areas you do committee work. Let's have a group solve a mathematics problem. If you can get yourself to allow committee work in your classroom, the classroom may tend to get very noisy as people are disagreeing on the best way to solve the problem. You who are from all-girls' schools have an advantage, because you don't have to deal with some of the issues of how to group students and whether girls and boys should be in the same group. Yes, I think we have to send this out to the whole world: Single-sex schools are good for girls.

Another thing I will mention is to have students present a math topic to the rest of the class. Again, notice how much I'm dealing with communication about mathematics. I am taking mathematics out of the realm of "Use it here and work on it alone, and I'll read it in the privacy of my room, and we'll know if you know it or not," to "Let's share these ideas about mathematics."

One other suggestion. If you want to have mathematics be something that is appropriate for girls to do, bring your wastepaper basket into class. Let them see what happens when you get a new textbook and start working through the problems. What we never show our students is that it took you twelve pieces of paper before you could even start the problem. That is important because young women want to be perceived in a favorable light. They want to be right. And you need to show that you take risks. "I don't know how to start this problem..." and show them that you didn't just know

it instantaneously. That is one of the myths about mathematics, that you have a "math mind"—you look at a problem and that the whole solution, in columns, comes out right in your mind. We need to share our inadequacies as mathematicians, the fact that we do have to sit down and work the problem. Those are the kinds of things that I think will help to encourage young women to go on in mathematics and science.

You will have some colleagues who will not be happy with a lot of the things that I have said. I was asked a question during the break, "Do we need to get the girls ready for the real world where it's going to be tough?" My response is no. Why don't we create for them a world in which they can function and be happy and learn, because then they can go out there and change that other world. Because if you make them just like everybody else, then there is no chance for changing what we are doing. So those are my feminist views. I use the word proudly. I think that feminist pedagogy has a lot to say about how we teach mathematics and science. ❖

Judith Jacobs'

Techniques to Address *Women's Ways of Knowing* In the Mathematics Classroom

- Use cooperative learning
- Incorporate writing activities

Reaction Papers

- Write down what you have learned
- Write down what you have questions about

Journals

- Keeping track of feelings as students learn mathematics as well as how they are doing mathematics.

Papers and Reports

- To connect students' interests
- Have students watch other students do mathematics

Interviews

- How did you solve this problem
- Have students present a math topic to the class,
- This takes mathematics out of the realm of "use it here and work on it alone, and I'll read it in the privacy of my room, and we'll know if you know it or not" ... and takes it to "let's share these ideas about mathematics."

Women's Ways of Knowing

Stage of Knowing	Statement
Silent Knowing	
Accept authority's verdict as to what is true	An inner voice expresses awareness that teachers think base angles ...
Received Knowing:	
Learn by listening; returns words of authority. Speaker is not source of knowledge.	"I know that base angles are equal because my teacher says so."
Subjective Knowing:	
Inner voice says "I only know" what I feel in my gut." Assumes there are right answers.	"I know that base angles ... Just look at them; they're equal."
Male version: "I have a right to my opinion." Female version: "It is just my opinion."	
Procedural Knowing:	
Voice of reason; begins to evaluate validity of argument	"I know these are equal, but maybe all base angles are not. I need proof."
Separate knowing; Looks to propositional logic; impersonal way of knowing.	"I know that it looks that way, but what about the triangles that other people looked at? Let's look at those too."
Connected knowing; Looks to what circumstances lead to perception; wants access to other people's knowledge.	
Constructed Knowing:	
Effort to integrate what is known intuitively and what other people know. Appreciates complexity of knowledge.	"Let's physically compare the angles." "Tell me why you think that base angles ..."

Shirley M. Malcom

Head, Directorate for Educational & Human Resources
American Association for the Advancement of Science

SHIRLEY MALCOM IS HEAD of the Directorate for Education and Human Resources Programs of the American Association for the Advancement of Science (AAAS). The Directorate includes AAAS programs in education, activities for underrepresented groups, and public understanding of science and technology. Dr. Malcom was head of the AAAS Office of Opportunities in Science from 1979 to 1989. Between 1977 and 1979, Dr. Malcom served as program officer in the Science Education Directorate of the National Science Foundation. Prior to this, she held the rank of assistant professor of biology at the University of North Carolina, Wilmington. Other work experience includes two years as a high school science teacher.

Dr. Malcom received her doctorate in ecology from the Pennsylvania State University; master's degree in zoology from the University of California, Los Angeles; and bachelor's degree with distinction in zoology from the University of Washington.

In 1989, Dr. Malcom received Humanitarian of the Year Award from the National Coalition of Title 1/Chapter 1 Parents, the National Medical Association's Scroll of Honor, and the Golden Gazelle from the Project on Equal Education Rights of the National Organization of Women Legal Defense and Education Fund. ◆

Science for All Americans

~ *Project 2061* ~

Shirley M. Malcom

DR. MALCOLM DISCUSSED THE MANY ROLES of career women today, the special needs of minority women, and the objectives of the 2061 Project. The excerpts that follow are from the summary of *Project 2061: Science for All Americans.*

Science for All Americans is about scientific literacy. It consists of a set of recommendations by the National Council on Science and Technology Education—a distinguished group of scientists and educators appointed by the American Association for the Advancement of Science—on what understandings and habits of mind are essential for all citizens in a scientifically literate society.

Scientific literacy—which embraces science, mathematics, and technology—has emerged as a central goal of education. Yet the fact is that general scientific literacy eludes us in the United States. A cascade of recent studies has made it abundantly clear that by both national standards and world norms, U.S. education is failing to adequately educate too many students—and hence failing the nation. By all accounts, America has no more urgent priority than the reform of education in science, mathematics, and technology.

But educational reform cannot simply be legislated. It will take time, determination, collaboration, resources, and leadership. It will take daring and experimentation. And it will take a shared national vision of what Americans want their schools to achieve. *Science for All Americans*—part of a AAAS initiative called project 2061—is intended to help in the formulation of that vision.

In preparing its recommendations, the national council drew on the reports of five independent scientific panels. In addition, the national council sought the advice of a large and diverse array of consultants and reviewers—scientists, engineers, mathematicians, historians, and educators. In all, the process took more than three years, involving hundreds of individuals, and culminated in the unanimous approval of *Science for All Americans* by the board of directors of the AAAS. ❖

Leslie Hiles Paoletti

Mathematical Sciences Education Board
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LESLIE PAOLETTI IS THE MATHEMATICS CHAIR of Choate-Rosemary Hall. A graduate of Bryn Mawr College, A.B. in physics, and Yale University, M.A.T. in physical science, Ms. Paoletti previously held positions in mathematics and physical science teaching at the "School Within a School" at Middletown High School, Middletown, Connecticut; Hanover High School, Hanover, New Hampshire; and Wilbur Cross High School, New Haven, Connecticut.

She was a 1991 Connecticut Finalist for the Presidential Award for Excellence in Mathematics Teaching, Woodrow Wilson Mathematics Institute Fellow (1987), and served on the Mathematical Sciences Education Board 1988-1991. She is Past-President of ATOMIC (Associated Teachers of Mathematics in Connecticut). Since 1984 she has participated in PIMMS (Project to Increase Mastery of Mathematics and Science). She has led numerous workshops and has had much of her work published. ❖

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A Report on Recent Research & Publications

~ *On the reform of teaching mathematics* ~

Leslie Hiles Paoletti

I WOULD LIKE TO THANK MY COLLEAGUES from Connecticut, Louise Gould and Ann Pollina, for their tireless efforts in organizing this exciting conference and especially for offering me the opportunity to participate. My comments today derive primarily from my work over the past three years as a member of the Mathematical Sciences Education Board—an umbrella group attached to the National Academy of Science, which is attempting to stimulate and coordinate reform in the teaching and learning of mathematics K-12 at the national level through various publications and close cooperation with the National Council of Mathematics, the Mathematical Association of America, the National Science Foundation, the Departments of Education and Energy, and at the local level through the State Coalitions.

Curriculum Standards and Teaching Standards

The specific content recommendations for the teaching and learning of mathematics in grades K-12 can be found in the NCTM document—the *Curriculum and Evaluation Standards* released two years ago. Recommendations for implementing these standards can be found in the *Professional Teaching Standards* released this spring. In addition to standards for teaching, this second volume contains standards for evaluation of teaching, for professional development of teachers, and for the critical ongoing support that is necessary. If you haven't purchased and read these two documents, I urge you to add them to your "required summer reading" list, and to whet your appetite, the conference is providing a copy of the summary of each of these documents for you. If you have these documents already, please pass your copy of the summary along to the most powerful person in your school—that is, powerful in helping you make the changes you'd like in the teaching and learning of mathematics—the department chair, the

principal, headmistress, chairman of the Board of Trustees, business manager, or head of the parents' group. It is extremely important that we move outside our own classrooms and into the total community. The necessity and effectiveness of such a total approach is clear both in the studies of education in Japan showing the effect of cultural attitudes on student expectation and achievement, and in our own country, from evaluation of the work of James Comer in the New Haven public schools—a model now being replicated nationwide.

Teachers Can't Do It Alone

So my first report from research: teachers can't do it alone, but we must take the initiative in gaining community support. The national agenda is complex, with problems from military expenditures and S&L bailouts to those of the environment and the homeless, and there are competing voices for budget and time in education itself—including sex education, health education, bilingual education, literacy, and handicapped access. However, mathematics and science education now occupies a prominent position. The President's Goals for Education which are shared by the National Governors' Conference and are included in a recently released "gold book", *Education 2000*, call for the United States to be number one in mathematics and science by the year 2000. In addition, the steps already taken by the mathematics community are now being seen as a model for reform in other disciplines; we have our act together. We have agreed upon standards. We have data which compare the performance of our students both nationally and internationally. And we can enlist the support of the wider community—administrators, parents, school boards, the media, and the politicians because mathematics and science education is seen as the key to our global economic survival. We may, of course, have our own personal goals—to empower our students to think critically and independently and to share the beauty of mathematics, but we cannot neglect the opportunity that economic concerns have given us to persuade the wider community to join us in our efforts. With our own students, we also need to personalize the importance of mastery of the concepts and skills of mathematics and science for their own futures beyond formal education. As author Kati Marton said in the recent Choate Rosemary Hall Commencement address which she shared with her husband Peter Jennings, "A woman without a craft or profession is one man away from welfare." We must take the lead in imparting a sense of urgency for greater mathematical proficiency for all—male and female, white, black, Hispanic, and Native American.

As we begin to share the ideas found in *Curriculum Standards* and the *Teaching Standards* with the wider community and to seek the support of a broader constituency to implement them in our classrooms, we must first ask ourselves how do we really know that the changes that we are advocating

really will provide better mathematics learning for all A summary of recent research which underpins national recommendations can be found in *Reshaping School Mathematics, A Philosophy and Framework for Curriculum*, published by the National Academy Press and for sale during the break and before and after dinner. Both the text itself and as the research cited in the extensive bibliography provide some preliminary data in answering these questions.

We must be able to answer three primary questions regarding change — why, what, and how.

Why Change?

The urgent need for revitalizing the teaching of mathematics is most eloquently stated in *Everybody Counts: A Report to the Nation on the Future of Mathematics Education*. Again, a summary has been provided for all participants, but I urge you to read the complete document and to share its contents throughout your community. Three out of four Americans stop studying mathematics before completing career or job prerequisites. And as we know, a disprorportionate number of these dropouts are minorities and women. Yet white males will make up only 15% of the projected new entrants to the workforce between 1985 and 2000. Most students leave school without sufficient preparation in mathematics to cope with college expectations for numeracy, on-the-job demands for problem solving or the understandings necessary to make informed decisions—on issues from undergoing elective surgery to locating hazardous wastes. As a result universities, the armed forces, and industry are all heavily burdened by the need to provide remedial education. Industry alone spends as much on mathematics education as the total amount spent in all schools, colleges and universities combined. We need a citizenry that is mentally fit to absorb new ideas, to adapt to change, to cope with ambiguity and complexity and to solve unconventional problems—not divide a 5-digit number by a 3-digit number or simplify a rational expression. The results released in the last two weeks by the National Assessment of Educational Progress indicate that while 98% of the nation's eighth graders can perform simple addition and problem solving with whole numbers, only 14% have achieved "average proficiency"—the ability to solve problems that involve fractions, decimals, percents, elementary geometry and simple algebra, and only in one state, Virginia, were there any students (only 1%) who could solve algebraic equations, as well as problems involving beginning statistics, probability, and reasoning involving geometric relationships. Indeed the media continually reminds us why we must change.

What Must Change?

Again, specific curriculum conztent is provided in the NCTM Curriculum and Evaluation Standards. However, more important is the most elusive goal—to develop in our students the ability to think and a belief and confidence in that ability so that they can become life-long learners. Not only must they know how to crank out numbers from a mathematical model, they must also know how to evaluate the assumptions inherent in the construction of the model and be able to use their results as well as new data to improve the model. In changing the "what" we must change parental beliefs. The math that was good enough for them is not good enough for their children. We must eradicate the false pride that some take in "not being very good in mathematics." We rarely hear anyone boast, "My family doesn't read." We especially must disabuse our society of the notion that mathematical proficiency is only for the elite, talented few. All students must possess the confidence to tackle unfamiliar problems and to know how to choose and where to find the mathematical tools necessary for solving them — at home and in the workplace. Finally, what we teach should not be limited to what we can easily and cheaply test.

How to Change?

A clear vision of effective classroom practice which promotes the learning of mathematics is provided in the first sixty-seven pages of the *Professional Teaching Standards*. Careful reading and discussion of the vignettes and commentary by teachers may be the most efficient and powerful way to transform the teaching of mathematics. Examples are given of selecting tasks which engage both the interest and intellect of students and provide opportunities for them to attain a deeper understanding and to seek connections between previous and developing knowledge. Research in both cognitive psychology and mathematics education confirms that true learning occurs as students actively assimilate new information and experience and construct their own meanings. The least effective method of teaching is the one still most prevalent in today's secondary mathematics classrooms — lecturing, listening, and imitating. Unfortunately, this is the method which current teachers, administrators, and parents experienced in their mathematics classrooms — and often the method which our students have experienced in earlier grades. So we must find ways to change widespread beliefs about accepted modes of instruction. In addition to the selection of appropriate tasks, teachers must learn to monitor what the *Standards* describe as "discourse" — ways of representing, thinking, talking, and questioning mathematical ideas. Mathematics is a powerful, efficient and internationally understood language. The physicist Richard Feynman eloquently reminded us, "If you want to understand nature, you must be conversant with the

language in which nature speaks to us." Certainly the beauty of computer-generated fractal images of natural phenomena gives further credence to the power of mathematics in describing the complexities of the physical world beyond the simple elegance of Newtonian mechanics. Moreover, mathematics is a language which helps us make sense not only of scientific phenomena, but also contributes to our understanding in almost every field, from economics to archaeology, medicine to music. We must be sure that we and our students use this language of mathematics in our classrooms in ways that are "sense-making" for all. Small group discussions, dialogues, individual student presentations, journal-keeping, as well as whole class discussions are all ways that students can develop a facility for mathematical discourse.

Major Obstacles

There have been two major obstacles to reform at the secondary level — standardized testing and antiquated teaching methods at the college and university level. I'm happy to report that progress is being made at the national level in confronting both of these issues. In April, the Mathematical Sciences Education Board sponsored a National Summit on Assessment, which followed four earlier regional meetings around the country. The chair of the steering committee was Secretary of Education Lamar Alexander. He brought with him to the conference President Bush, who stated, "As you consider your principles, goals and actions for math assessment, let me just ask you to keep a few questions in mind. Consider what it means to be the best in the world and the kind of balance our students will need between theoretical math and practical applied skills for life. How can we create tests to ensure not just that our best students are as good as any in the world, but that our average students achieve world-class status? How can we emphasize testing that encourages better teaching? That doesn't weed kids out, but develops better math skills for all?" The presidential rhetoric is a beginning. The challenge to the mathematics community — to develop alternative assessment which is consistent with the changes in teaching we are advocating — is formidable. But a beginning, to develop open-ended assessment instruments as well as accompanying scoring rubrics, has been made in sizeable projects in both California and Connecticut.

The second area of concern which impacts on secondary mathematics, inadequate teaching in higher education, is being addressed by the MAA, the MSEB, and the Board on Mathematical Sciences. A joint committee on "Mathematical Sciences in the Year 2000" has just released its report, *Moving Beyond Myths: Revitalizing Undergraduate Mathematics*. The accompanying *Action Plan*, which is also available for you at the end of this session, contains recommendations that are as relevant at the secondary level as they are at the undergraduate level. Goals include "Full utilization of the mathematical potential of women, minorities, and the disabled." College

faculty are implored to "learn about learning, think as deeply about how you teach as what you teach, involve students actively in the learning process"...and my favorite "Teach the students you have, not the ones you wish you had."

Obstacles to Change

In looking at obstacles to change, we should also look at the resources that are now available to us. The technological revolution, especially the widespread availability of the handheld calculator and personal computer, not only forces us to consider why, what, and how the teaching of mathematics must change, but it provides us with opportunities which we are just beginning to explore. Rather than the false notions that the calculator eliminates the need for teaching elementary mathematics, and the graphing calculator and graphing utilities and symbolic manipulation programs available on the computer eliminate the need for much of secondary and undergraduate mathematics education, these instruments allow us to greatly enrich our teaching. We can explore real-world problems rather than limit ourselves to contrived situations in which systems of linear equations have integral solutions or, even more unlikely, where an appropriate mathematical model is a quadratic equation which is factorable. The visual power of the graphing calculator and computer greatly facilitate students' conceptual understanding. Whether exploring number patterns on a calculator, testing hypotheses with the geometric supposer, or discovering the relationship between the parameters of a function and its behavior using a graphing utility, technology allows the study of mathematics to be an engaging, creative activity. Mathematics is a science, filled with observation, experiment, discovery and conjecture. These activities have always been a part of the practice of mathematics. Shouldn't they be a part of the teaching and learning of mathematics as well?

Who?

In addition to the questions of why, what and how, we must be sure that everyone understands the answer to the "who." Although we are particularly concerned about young women, our concern is part of the greater concern that all students develop proficiency in mathematics. Only 39% of high school students take four years of mathematics. Although the College Board published on the front page of its newsletter, as a startling result, its study which showed that students who take more mathematics courses score higher on the mathematics section of the SAT, we all know that a major factor in what some perceive as a "gender difference" in mathematical ability is in fact no more than a gender difference in mathematical experiences.

In Conclusion

In closing, I would like to return to my first observation—that teachers can't do it alone. And indeed, we as mathematics teachers can't do it alone. We must first look to our colleagues in our own discipline for advice and support. We should also look to our colleagues in the physical and social sciences, as we begin to appreciate the teaching of mathematics as a science, for their help in developing exploratory activities, for data sets, and for teaching guidance in laboratory investigations as well as in involving students in discussions and debate. In many ways the reform movement in mathematics most closely parallels the work of the National Writing Project. Just as we are de-emphasizing rote learning of skills and math facts and refocusing our energy to developing understanding of concepts, so too are the writing teachers de-emphasizing the mechanical issues of spelling and punctuation and focusing on the clear expression of ideas. As mathematics is truly also a language, we can learn much from our colleagues who teach language arts. Their experience in assessing writing can be especially valuable to us. Mathematics, like writing, is a process—not a set of facts. What we believe mathematics to be critically determines what we teach and how we teach. These ideas are not new. Descartes called mathematics the "Science of Order." Lynn Steen has written about the "Science of Patterns." *Science for All Americans* extends this definition to the "Science of Patterns and Relationships." As women teaching women, these changes are especially significant as we place greater emphasis on communication, not silent acquiescence; on cooperative learning and sharing ideas, not competition; on the value of mathematics in helping us understand the real world and solve human problems, not rote memorization of abstract definitions; on complex, integrated tasks, not hundreds of discrete exercises which are to be completed in a minute or less; and on alternative approaches, not a single right answer. Changing our class environments should reduce the fear and embarrassment that girls and minorities too often feel, and promote risk-taking, self-reliance and empowerment. Even the fine motor skills which are demanded for use of the computer and calculator are skills at which girls have been shown to excel at an early age. Our special attributes and cultural values as women are uniquely suited to enable us to lead in the reform of mathematics education. Determining the best and most effective curriculum (content, pedagogy and assessment) is not a task that we will do at this conference or in our five-year plan at our schools. As with any good problem, solutions lead to new and better questions. As we search for more effective learning, we can model the exciting, dynamic mathematical process we are trying to impart to our students. As female teachers we can perhaps best model the supportive, collaborative structure necessary for this unending quest! I look forward to learning with you and from you, not only at this conference, but also in the months and years ahead. ❖

Bibliography

National Council of Teachers of Mathematics. *Professional Standards for Teaching Mathematics*. Reston, VA: The Council, 1991.

National Research Council. *Everybody Counts: A Report to the Nation on the Future of Mathematics Education*. Washington, DC: National Academy Press, 1989.

National Research Council. *Moving Beyond Myths: Revitalizing Undergraduate Mathematics*. Washington, DC: National Academy Press, 1991.

National Research Council. *Reshaping School Mathematics: A Philosophy and Framework for Curriculum*. Washington, DC: National Academy Press, 1990.

U.S. Department of Education. *America 2000: An Education Strategy*. Washington, DC: U.S. Department of Education, 1991.

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She has published many articles on education of the gifted. She has served as a consultant to a number of programs for the gifted as well as reviewing texts for several professional journals.

Her report consisted of sharing two of her recent research projects. Provided by her are the abstracts of the two papers cited, which should appear in publications soon. ❖

Highlights of Gender Differences

~ *In classroom participation and achievements in AP calculus* ~

Rena F. Subotnik

RATIONALE:

Gender differences appear to be greatest in math and science achievement in the gifted population.

DEFINITIONS:

Gifted—master the given curriculum at high level.
Talented—capable of extending an area of study.

ASSUMPTIONS:

- Notion of gender differences is self-perpetuating, but can be corrected.
- Acknowledging that there are women's ways of knowing is fine, but girls should be able to choose.
- Everything we learn from gender-difference studies benefits boys as much as girls.

CONCLUSIONS DERIVED FROM CALCULUS STUDY:

- Keep high expectations and check homework.
- Test for preparedness.
- SAT-M/PSAT-M are not perfect predictors.
- Let kids try to test into higher level math.
- Operate small groups or dyads, sometimes single sex.
- Experiment with note taking techniques.

Five Critical Areas Affecting Retention & Attrition of Talent in Science:

- Method used for choosing an undergraduate institution to attend.
- Degree of confidence in a unique talent.
- Availability of tacit knowledge from mentors.
- Availability of "lower" track options.
- Com.nitment from federal government and the science establishment.

An Experiment Involving Advanced Placement Calculus Classes

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Abstract

This study* focuses on gender differences between gifted males and females enrolled in Advanced Placement calculus, a college level course taught at the secondary school level. Subjects (N=136) were students at two sites, one a publicly funded laboratory school for gifted students, and the other a highly selective girls' independent school in the same metropolitan area. The study objective was to examine gender differences in classroom interaction patterns and achievement in advanced high school mathematics. Subjects' scores on a calculus readiness test, taken at the beginning of the experimental period, and the SAT-M served as predictors of achievement on the Advanced Placement (AP) exam. Voluntary rather than teacher selected participation in the classroom served as a predictor for success on the examination as well. Neither the number of questions generated by students, the number of responses called out, the number of teacher appointed responses, nor assignment to an experimental single-sex section at the laboratory school were statistically useful predictors for AP scores. There were no gender differences in AP calculus scores between male and female subjects in this study.

- *The author dedicates this study to the memory of Patricia Lund Casserly, an inspiration and a mentor.*

* *This study was funded by a Research in Teaching and Learning Grant of the National Science Foundation.*

Retention and Attrition of Science Talent: A Longitudinal Study of Westinghouse Science Talent Research Winners

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This paper will appear as an article in an upcoming issue of The International Journal of Science Education.

Abstract

Ninety-four winners of a highly prestigious American science talent search were followed longitudinally from the time they received the award in 1983 to the present. The subjects, 57 male and 37 female, were mailed a 20-item questionnaire designed to monitor changes and consistencies in the values underlying decisions to pursue or not pursue science as a field of study, as an area of research, and/or as a possible career. By 1988, 41% of the females and 12% of the males had dropped out of science. Based on the results of the questionnaire and follow-up interviews, those who have remained in the science track were more likely than those who dropped out to have been supported in their efforts to carry out research projects, given resources to engage in research, and provided mentoring at least equal to that provided during the high school years, including exposure to tacit knowledge needed to succeed in the world of scientific research. ❖

Gifted Girls	Talented Girls
May or may not have passion to jump hoops Like to do well in school Likely to become professionals	Like creative challenge May or may not have been labeled gifted Likely to go into academic or other research
Needs	**Needs**
Recruit into high level courses Monitor their work Provide realistic information about careers	Access to equipment, time, people Good advice about schooling, especially post secondary

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JOAN COUNTRYMAN IS ASSISTANT HEAD for Academic Planning at Germantown Friends School. She is the daughter of a mathematics teacher, the mother of a mathematics teacher, and has taught mathematics herself for 21 years. Her current responsibilities include academic planning for K-12, with a special interest in the impact of technology on education, especially for younger children.

She holds a B.A. degree from Sarah Lawrence and an M.A. in teaching from Wesleyan University. She attended Yale University doing graduate work in city planning, and participated in a Fulbright at the London School of Economics while she was studying city planning. She has taught for the Woodrow Wilson Foundation summer institute for high school teachers of mathematics. ❖

Is Gender an Issue in Math Class?

~ *Perhaps it's time to change the subject* ~

Joan Countryman

WHAT WOULD HIGH SCHOOL MATHEMATICS BE if women were central to our teaching and our thought? How would the metaphors change? Would we still speak of 'mastery', 'drill', and 'attacking problems'? Would the emphasis still be logic, abstraction, rationality, and right answers, or would it become construction, exploration, intuition? Would it shift from product to process? From exclusion to inclusion? From secrets and mysteries to shared visions and stories? What happens to our concepts, our theories and methods when we move to an inclusive curriculum?

There are no simple answers to these questions, though they are central to every discussion of curricular change, especially during a time of revision in the mathematics curriculum. We need to think critically about the way that mathematical knowledge is constructed, and about the assumptions that are occasioned by our teaching. What do our students learn as we teach them mathematics? What do we learn from them?

Education is always in crisis.[1] The solutions offered for today—teacher evaluation, teacher retraining, assessment generally and testing in particular, copying the Japanese or the people of any other industrialized nation—ignore the deep alienation of students, indeed of most people, in the case of mathematics. What does it mean to become educated? What does it mean to learn mathematics? Who decides? Educated for what? About whom? In a paper called *Denying Difference: The Continuing Basis for Exclusion in the Curriculum*,[2] Margaret Anderson argues that the term "socialization", often used in a discussion of gender or racial differences, fails to acknowledge the power of the dominant group. Be it male, or white, and most often the dominant group is white and male, the group maintains its view while claiming that it has no view, that the group has no perspective. It is the same reasoning that claims that the words 'human' and 'male' are synonymous.

What happens when we put women at the center of the construction of our courses? How might the curriculum be transformed? Suppose we go beyond inserting a few women here and there. What if we take up Peggy McIntosh's challenge and ask: What is the core of my discipline? How would mathematics have to change to reflect the fact that women are the majority of the world's population? Peggy McIntosh's *Phase Theory of Curriculum Reform*[3] provides a structure with which we can examine mathematics programs. The theory includes five interactive, often simultaneous phases:

The Phase Theory of Curriculum Reform

- *Womenless, White Male Mathematics.* Until recently, most textbooks and many math courses looked like this.
- *Women Injected into the Curriculum.* Women mathematicians, heroines, worthies, like Sonya Kovaleska and Ada Lovelace, become enrichment material. They are described as special, and different from the mass of women. The problem with this phase is that it takes the dominant group experience as the organizing framework for the discipline. Mathematics remains what white men do, along with a few token women. This method asks no fundamental questions of the discipline. Although discussing women mathematicians might motivate a few students, the models of excellence are those of the dominant group.
- *Women as Victims.* This phase emphasizes discrimination. Why don't girls win the math contests? Why are there not pictures of women in the textbooks? Who decides on the subject of the word problems? Sexism and racism are topics of study, but women (and other excluded groups) become "the problem". Who pities whom? Now this phase may be necessary, but it is also depressing. Students become dispirited, then resistant, then angry. Margaret Anderson reminds us that this is a disabling phase because it erodes our ability to withstand victimization and leads to bad classroom dynamics as students compete for victim status. "Men are victims, too." they whine.
- *Women's Mathematics.* This phase uses women's experience as the lens through which we view the world. It engages emotion *and* rationality, intuition *and* logic. We move away from examining oppression toward examining privilege, revealing how the structure affects everyone, not just the victims. We raise new questions, and make new assumptions. We study not just others, but ourselves. We begin to tell our stories, and listen to the life histories of others. It is the stage of mathematics autobiographies, and stories like Sonia Sanchez' *Norma*.[4] We ask each other what we can learn about math and science from our experience.

- *Reflective Mathematics.* The words to describe the fifth phase are more elusive. I find it hard to say what the transformed mathematics curriculum will be, or how we will achieve it. Surely in this phase mathematics will help us see ourselves and others in connection with the world.

Mathematics in Secondary Schools

I want to use this structure, the five phases, to examine the current state of mathematics in secondary schools. It is important to keep in mind that the phases are interactive. Most of us probably work in all of them at one time or another, in some of them simultaneously.

My examples are drawn largely from my own teaching, from observations, student writing, and student work. For twenty-one years I have taught secondary mathematics at Germantown Friends School in Philadelphia. My students are a diverse lot—generally middle and upper middle class, black, brown, white, female, male, all headed for college, some enthusiastic about mathematics. I have taught them algebra, geometry, precalculus, and calculus. They have come to see me as the strange person who requires that they write about learning mathematics.[5] Here are some things they say and do.

Phase One: Womenless Mathematics

Anyone who doubts that gender is an issue in the mathematics classroom need only listen to the following comments from senior calculus students:

> In elementary school I was a very good math student and always found myself working on math level tests much higher than those of my classmates. My teachers pushed me very hard, and I often got private lessons with the teacher while the rest of the class was working on other problems. I remember being proud of being the only third grader in the school to know long division." (Chip)

> "My earliest math memory is with failures. [In a new school] I didn't know how to multiply and the rest of the class was doing multiplication problems. I struggled to memorize the tables but I could not understand the concept.... (Katy)

Boys speak with confidence, girls do not. Among even my strongest students, unless I intervene, mathematics is seen as part of the male domain. The National Council of Teachers of Mathematics reports in its *Curriculum and Evaluation Standards* that current statistics indicate that those who study advanced mathematics are most often white males. This message is not lost on students, for more often than not they see a mathematics that contains neither women nor the concerns of women.

When asked to comment on their experience this is what they say:

> I think that I am not a mathematical person. I enjoyed the class, but I really don't think I learned anything except how to flunk with dignity...I think that this is a hard course. I tried, but my best just didn't make it. I am glad that I stuck with it though. I really don't care for integrals, limits, etc. (Molly)

Phase Two: Women in Mathematics

In 1937 Eric Temple Bell wrote a book, now back in print, called *Men of Mathematics*. In the 1970s Lynn Olsen wrote a corrective *Women in Mathematics*, a collection of biographies of women. Jacqueline Dewar of the mathematics department at Loyola Marymount University in Los Angeles has developed a college level course for liberal arts students that includes the study of the biographies of thirteen women mathematicians, four working in the twentieth century.[6] The course fulfills a science core requirement for liberal arts students and is recognized as an elective for the women's studies program. The course is designed to: give non-science majors new insight into mathematics as a creative art and science; give students some experience of the kinds of investigations that make mathematics compelling to mathematicians; improve attitudes towards mathematics; and change the impression that math is a male domain. The workload is substantial — homework, weekly quizzes on mathematics, two papers, two hour exams and a final, and a special project. The latter might be research on a mathematics topic, a program of study to improve skills, or a sociological study of mathematics education. The instructor describes getting to know students on a more personal level than is usual in a typical mathematics class. In the evaluations students write:

> I have enjoyed the class tremendously. Now I don't hate math anymore like I did when I entered college.
>
> I'm not as afraid and seem more relaxed taking math related exams.
>
> I think now that anyone can learn to do math—it just takes some people longer.

But listen to this comment from Jacqueline Dewar:

> There are two factors that make teaching this course difficult. One is that selecting interesting mathematical topics and presenting them at a level accessible to a group of students with a wide range of mathematical backgrounds is a challenge. The other has to do with the one or two males who enroll. It is not easy to get them truly engaged in the discussions.

I REPEAT: The problem with this phase is that it takes the dominant group experience as its organizing framework. Mathematics remains what white men do, along with a few token women. No fundamental questions are posed about the discipline.

Phase Three: Women as Victims

The depressing phase.

Although girls start school ahead of boys in reading and computation, by the time they graduate from high school, boys have higher SATs in both.

- Girls are less likely to take math and science even when they show ability in those areas.
- Girls are more likely to believe they are incapable of doing math and science, and to avoid it in high school and college.
- Girls attribute failure to lack of ability. Boys attribute failure to bad luck.
- Men speak more often and more often interrupt women.
- Listeners recall more from male speakers even when style and content are identical.
- Women participate less actively in conversation.
- Women question rather than declare (*kind of... I guess... is it?*).

Documenting failure, exclusion, and discrimination is easy, but the risk, as Anderson argues in "Denying Difference" is that we become the problem, and we also begin to look pathetic. In *Counting Girls Out*, a book based on the research of the Girls and Mathematics Unit of the University of London Institute of Education, Valerie Walkerdine points out that in their research:

> While we established that girls were felt to lack something, even when they were successful, it seemed equally and positively that boys were felt to possess the very thing that girls were taken to lack. It would seem easy to prove or disprove this by recourse to empirical evidence. However, we...argue that things are not so straight-forward. Girls are still considered lacking when they perform well and boys are still taken to possess something even when they perform poorly. In other words, we need carefully to examine the relationship between evidence and *explanation.*[7]

What is the problem of women and mathematics? Is there something wrong with women, or is there something wrong with mathematics? In truth, most people see themselves as victims when they talk about learning mathematics. If you don't believe me just introduce yourself as a math teacher at a party and watch how strangers respond to you. There is good evidence that eighty percent of the population of this country hates math-

ematics, and few seem embarrassed to say so. No one really wants to be seen as a victim though, and that makes me uncomfortable with the Math Anxiety approach to this problem.

Phase Four: Women's Mathematics

Phase Four is the stage where we begin to tell our own stories. For the last two years I have taught a precalculus course to eleventh grade students using the *Contemporary Precalculus Through Applications* materials written by the mathematics department at the North Carolina School for Science and Mathematics, and recently published by Janson Publications. The teachers who developed this curriculum point out that too often the mathematics that students experience at school consists mainly of algebraic statements, followed by algebraic manipulations, leading to solutions. And nothing more. In the real world, these are only the middle steps in a broader and considerably more interesting process.

A situation might lead to the statement of a problem, which in turn might lead to a mathematical model, then an algebraic statement, algebraic manipulation, perhaps a solution, interpretation of the solution in a mathematical context, and finally interpretation of the solution in the real world. The course has six spiraling themes: **mathematical modeling**, the use of mathematics to model a wide variety of phenomena; **computer and calculator as tools**, with the de-emphasis of pencil and paper techniques; **application of functions**, illustrating how functions serve as a bridge between mathematics and the real world; **data analysis**, drawing information (usually values for variables) from observation; **discrete phenomena**, including finance, sampling and polls, population growth, etc.; and **numerical algorithms**, particularly the use of numerical methods to solve interesting and motivating problems not easily solved by elementary techniques. The course is designed to encourage students to approach mathematics in new and innovative ways.

My students find the course compelling. To show what I mean I will quote from final papers written by four students, all of whom began the year thinking writing and mathematics were totally unrelated subjects. The assignment was to write a paper describing their exploration of a topic of interest to them. The investigation should involve data analysis and modeling.

First some excerpts from Hanna's paper:

> There are many stereotypes about mathematics: children do not like it, boys are better at it and so on. Many of these stereotypes do not hold true in the classroom. The goal of this project was to see if there is a correlation between age or sex or the child's attitude toward math and how well first grade students were able to transfer mathematical knowledge of three digit addition and subtraction of money.

A group of twenty-two first grade students was given a questionnaire which provided the data of this project. The questionnaire asked if the children liked math and if they found math hard or easy. The students had completed surveys like this one in the past and were accustomed to the format. They were also uninhibited about telling their true feelings. The survey then had ten problems of addition and subtraction of money (dollars and cents). The children have been doing three digit addition and subtraction for months using only numbers without the dollar signs and the decimal point. Their teacher expressed her confidence that all of the children would score 90% or higher on a simple test of addition or subtraction problems. When the signs were added, however, only twelve children were able to score a 90% or higher.

Hanna found that while all the children reported that they liked math, girls scored lower than the boys on the test. She poses these questions: If the children have the same attitude about math, then why is it that they score differently on tests? Are there teaching methods that would be better for these [still enthusiastic] young females?

Next, excerpts from Sophie's paper:

BIOGRAPHY OF THE QUESTION:

The problem gives a data set and asks us to fit a model to it, and use the model to find another point for the data set. The data given are the speed of a runner and the time it takes that runner to finish the race.... I decided to work on this problem because, as a runner, the topic interested me, and because I deeply enjoyed working the other data analysis problems in the textbook.

...Thus my model works well! It is accurate enough to predict values within a reasonable doubt, and the residuals are all relatively small. My model shows that as time increases speed decreases, or that there is a direct relationship between the log of the time and the runner's speed, this relationship, too, has a negative slope. To me, as a runner, this data is reassuring. Being a distance runner, I have a different perception of speed than a sprinter would, yet I do not think slow is fast. This model tells me that as time and distance go on, the decreases in my speed will get smaller and smaller.... Some further questions that occur to me...

Sophie goes on to speculate about 60 meter sprints and ultramarathons, the relationship between speed and time at different age levels and in different countries.

Eben wrote *Taking Stock of the Situation: Data Analysis on the Dow Jones Industrial Market and the Underlying Economic Factors.* He begins:

A complete answer to all my questions about this project just might solve a lot of the United States' problems! I looked upon it, however, as a way to work and look at graphs where there is no set answer, but from which an outsider might get an idea of the general relationship between the stock market and the economy. It became almost a drawing project for me, looking at balance, the shapes of line, having an inner vision that was new to me in mathematics. My grandmother would argue that this project is purely one about probability; my mother, who subscribes to the gamut of prediction and analysis, would suspect laws and definable occurrence.... Very broadly, is our society based on chaos or foreseeable trends?

One of Eben's models looks something like this: $f(x) = 70\ X^{.22} * abs(sin(.33(x+1) - 2.5))$.

Finally, from Jonathan:

Since I have been very interested and involved in physics, I wanted to find a topic for my final paper that would in some way investigate some of the principles we were studying in physics. I also wanted to study empirically one of these topics; that is, to take my own data and develop equations based not on a textbook but on my own data-taking and analysis. For these reasons, I decided to examine the behavior of different balls as they bounce, specifically to establish a relationship between the height from which a ball is dropped and the height that it then bounces. While I worked on this project, I also became interested in the time over which a ball continues to bounce, and I added my investigations in this area to my paper.

Jonathan finds an exponential function to describe the relationship between the height from which the ball was dropped and the time to stop bouncing. He comments that few equations in Newtonian physics involve logarithms, for most relations are either direct, inverse, or quadratic. He also raises questions about testing balls with different masses, volumes, and materials.

I have quoted at length from these papers because they demonstrate what I mean by phase four mathematics: emotion and rationality, students beginning to use mathematics to study their own questions, telling their own stories. What can they learn about math and science from their experience?

Phase Five: Reflective Mathematics

I said earlier that I did not have the words to describe phase five mathematics. That is true, but to give a sense of what it might be, I shall end

with quotations from two contemporary mathematicians, Marjorie Senechal, Professor of Mathematics at Smith College, whose field of research is mathematical crystallography; and Phillip Davis, Professor of Applied Mathematics at Brown University, whose work is in numerical analysis and the philosophy of mathematics.

In a new volume of essays on teaching mathematics for the twenty-first century, Professor Senechal discusses *Shape*.[8]

> Just as Shakespeare is not sufficient for literature and Copernicus is not sufficient for astronomy, so Euclid is not sufficient for geometry. Like scholars in all times and places, Euclid wrote about the concepts of geometry that he knew and that he could treat with the methods available to him. Thus he did *not* write about the geometry of maps, networks, or flexible forms, all of which are of central importance today.
>
> Shape is a vital, growing, and fascinating theme in mathematics with deep ties to classical geometry, but goes far beyond it in content, meaning, and method. Properly developed, the study of shape can form a central component of mathematics education, a component that draws on and contributes to not only mathematics, but also the sciences and the arts.
>
> Like many other important concepts, "shape" is an undefinable term. We cannot say precisely what "shape" means, partly because new kinds of shapes are always being discovered. We assume we know what shapes are, more or less: we know one when we see one, whether we see it with our eyes or in our imaginations.
>
> But we know much more than this. We know that shapes may be alike in some ways and different in others. A football is not a basketball, but both are smooth closed surfaces; a triangle is not a square, but both are polygons. We know that shapes may have different properties: a triangle made of straws is rigid, but a square made of straws is not. We know that shapes can change and yet be in some way the same: our shadows are always *our* shadows, even though they change in size and contour throughout the day.
>
> In the study of shape, our goals are not so very different from those of the ancient Greek philosophers: to discover similarities and differences among objects, to analyze the components of form, and to recognize shapes in different representations. Classification, analysis, and representation are our three principal tools. Of course, these tools are closely interrelated, so distinctions among them are to some extent artificial. Is symmetry a tool for classifying patterns or a tool for analyzing them? In fact, it is both...

SS

I look forward to a day when classification, analysis, and representation are listed as the principal tools of the elementary mathematics curriculum.

At a recent conference on mathematics as a humanistic discipline, Professor Davis argued that while tradition puts mathematics in the scientific orbit, he believes that it lies in both humanistic and technological cultures.[9] He explored the meaning of the term *humanistic mathematics* by comparing some of the qualities of mathematics with those of literature. His list includes:

METAPHOR - Applied mathematics is especially metaphorical. A model is indeed a metaphor, and we need to be careful not to confuse the mathematical creation with the real object.

AMBIGUITY - The genius of poetry is its ambiguity. Despite its emphasis on precision, math also has ambiguity. For example, which is the true geometry?

AESTHETICS - Think of high school geometry. Eliot said that the meaning of poetry cannot be written down. The same can be said for mathematics.

PARADOX, MYSTERY, AWE - Think of the current interest in fractals.

OUTCOME - Structure; the sense of completion when things come out right.

CATHARSIS - Passing from intuition to proof. Modern literature is anticathartic; so is mathematics.

Professor Davis went on to point out the contributions of mathematics and poetry to philosophy. ($\sqrt{2}$, and theology, and ∞). Both mathematics and poetry provide escape from the troubles of the world. This is not to ignore the fact that mathematics has both redemptive and destructive features. One has only to think of patriot missiles. To be humanistic in mathematics means to refocus on teaching. Mathematics possesses an awesome power to change our lives, but we must subject it to constant scrutiny.

So, here is mathematics challenging us to see ourselves and others in connection with the world. Anne Berthoff, in an essay about journal writing called *Dialectical Notebooks and the Audit of Meaning*, speaks of science and poetry:

> The more we can see in common between science and poetry, the easier it will be, then, to value the peculiar strengths of each as forms of knowledge. Those learning to write can learn a great deal from seeing how the scientist's work is related to the poet's and how what the historian does is close to both. As thinkers and formers, interpreters and creators—as composers and writers—they are all naming the world, bringing ideas to bear on what they are naming and imagining and hypothesizing and transforming.[10]

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In Conclusion

Here, finally, is one of my students, writing in her math journal, naming the world:

> I like talking about rational and irrational numbers, and about how pi is irrational and how that shows that nothing you ever, ever measure is truly correct, it usually just suffices.
>
> The difference between understanding and knowing is an enormous one. What I mean is that it is relatively easy to say "ah, yes" strange attractors, they look like butterflies or horseshoes, but what does that mean? So I am struggling with how much I can put into something in order to truly know it (i.e. be able to replicate, explain, know why, not just how). It's really hard, but there is so much in the math that can be so much more than just numbers. Especially in chaos; it mirrors so much of the real world. It seems to affirm it, because it takes the irregularities usually as white noise and randomness, the irregularities people talk about when they talk about mistakes and inaccuracies, it takes them and turns them into a science of unpredictability.
>
> A simple function can produce chaotic patterns if its output is used as input in a recursive loop. Depending on what the primary input is, different cycles of chaos and order arise at different times. What this means I'm not sure. Is it that the numbers obey some kind of rule in relation to each other for a while and then poof! it's gone? That is really strange. There are all these key phrases that I keep reading and hearing. Gypsy moth pop., fluid flow, nonlinear equations (apparently very discouraging things, related to fluid flow), Julia sets, basins of attraction (where do these things exist? on a graph? an imaginary no. graph? outer space?), osculating systems. There is such an amazing amount of creation from just some numbers, some lines, some dots. ❖

Applying Peggy McIntosh's Phase Theory of Curriculum Reform

by Joan Countryman

Phase One: Womanless White Male Mathematics.

Textbooks and many math courses looked like this until recently.

Phase Two: Women Injected into the Curriculum

Woman mathematicians such as Sonya Kovaleska and Ada Lovelace become enrichment material. They appear as heroines. They are described as special and different from the mass of women. This position takes the dominant group experience as the organizing framework for the discipline. Mathematics remains what white men do, along with a few token women.

Phase Three: Women as Victims

Why don't girls win the math contests?

Why are there not pictures of women in the textbooks?

Who decides on the subject of the word problems?

Students become dispirited, then resistant, then angry.

Phase Four: Women's Mathematics

This phase uses women's experience as the lens through which we view the world.

It engages emotion, and rationality, logic and intuition.

We begin to tell our stories and listen to the life histories of others.

We include the writing of mathematics autobiographies in our classroom activities.

We share stories like Sonia Sanchez' "Norma."

We ask each other what we can learn about math and science from our experience.

Phase Five: Reflective Mathematics

This phase is elusive but mathematics will help us see ourselves and others in connection with the world.

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References

1. Father Stanley Bezuszka of Boston College has a set of apocryphal documents tracing the history of teacher and parental complaints about students' use of calculators, ball point pens, fountain pens, ink, paper, and bark. It reminds me of Raymond Williams' insight that nostalgia stretches back to the garden of Eden. But then Eve made the garden of Eden problematic.

2. Margaret Anderson. Copyright 1987, Memphis State University, Center for Research on Women.

3. Center for Research on Women, Wellesley, MA.

4. Sonia Sanchez, *Homegirls & Handgrenades*, New York, Thunder Mouth Press, 1984.

5. My work in this area is described in William Zinsser's *Writing to Learn*, New York, Harper and Row, 1988, and in my *Writing to Learn Mathematics*, Heinermann Boynton-Cook, forthcoming.

6. Jacqueline M. Dewar, "Mathematics: Contributions by Women." Department of Mathematics, Loyola Marymount University, Los Angeles, CA 90045.

7 Valerie Walkerdine and The Girls and Mathematics Unit, *Counting Girls Out*, London, Virago, 1989.

8. Lynn Arthur Steen, editor, *On the Shoulders of Giants*, Washington, DC. National Academy of Science, 1990.

9. cf C.P. Snow, *Two Cultures*.

10. In Toby Fulwiler, editor, *The Journal Book*, Portsmouth, NH . Heinemann Boynton/Cook, 1987.

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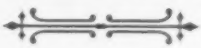
PART TWO

Small Group Workshops

Introduction



THE THIRD PHASE OF THE CONFERENCE made use of the most undervalued resource in the struggle for gender equity ... the classroom teacher. A day of small group workshops with educators selected for their innovation and experience was followed by two sessions of brainstorming and exchange.

Each participant selected two of the following two-hour workshops. Dr. Dorothy Buerk, Associate Professor of Math Education, Ithaca College, provided writing activities to approach the topic "Women's Metaphors for Math, Connecting with Mathematics: How We Do It, How We'd Like to See Our Students Do It". Cornelia Brunner and Margaret Honey, Research Scientists, Bank Street College of Education, used creative constructions to approach the topic "Gender and Technology: Exploring the Question of Technological Imagination". Caroline Hopp, Doctoral Student in Curriculum and Instruction, University of Wisconsin, shared the latest research on "Instruction with Cooperative Small Groups: Does It Really Influence Individual Learning". Dr. Claudia Henrion, Assistant Professor of Mathematics, Middlebury College, used imaginative construction of the mathematics needed by different cultures in past and future times to explore "Placing Mathematics in a Cultural and Historical Context". Dr. Barbara Lazarus, Associate Provost and Adjunct Associate Professor of Educational Anthropology, Carnegie-Mellon University, shared her findings about how to keep female students in the pipeline through her presentation "The Math, Science and Engineering Major in College: What Turns Them Off and How Can We Retain Them?" Dr. Jan Serie, Associate Professor of Biology, Macalester College, introduced recent computer software to share "Techniques of Teaching Science that Accommodate Diverse Learning Styles".

The text in the following sections has been transcribed from the actual workshop sessions. It reflects the conversional format of the workshops and the impromptu discussions that arose. ❖

Dorothy Buerk

Associate Professor , Math Education, Ithaca College

DOROTHY BUERK IS AN ASSOCIATE PROFESSOR in Math Education at Ithaca College. When she completed her doctorate in Math Education at the State University of New York at Buffalo, her thesis was entitled "Changing the Conception of Mathematics Knowledge in Intellectually Able, Math Avoidant Women". Dorothy has been working to change that conception ever since. At Ithaca, she teaches precalculus courses in math for math avoidant students, a writing seminar in mathematics, and coordinates a preservice program in secondary mathematics.

She has contributed to such publications as *For the Learning of Mathematics, Journal of Education, Humanistic Mathematics Newsletter* and the MAA notes *Using Writing to Teach Mathematics*. Her work currently in press will appear in the collection *Essays on Humanistic Mathematics*. ❖

Women's Metaphors for Math

~Connecting with mathematics; how we do it, how we'd like to see our students do it ~

Dorothy Buerk

OUR WORKSHOP TITLE TODAY IS "Connecting with Mathematics." In this workshop we will discuss themes from the metaphors for mathematics written by high school and college women in mathematics classes. Prior to the workshop, participants will receive a protocol for gathering metaphors from their own classes and may want to bring some examples of their own to share. We will talk about our views of mathematics, the views we would like our students to hold, and the views held by our math avoidant students. We will discuss classroom strategies to develop views of mathematics that empower our students as active, confident mathematics learners. We will use as a background the notions of connected knowing and connected teaching developed by Belenky, Clinchy, Goldberger, and Tarule in *Women's Ways of Knowing*.

REQUIRED READING FOR THE WORKSHOP: "The Voices of Women Making Meaning in Mathematics" Dorothy Buerk. Journal of Education, Vol. 167, Number 3, 1985.

WHAT FOLLOWS ARE EXCERPTS FROM THE DISCUSSION.

We have found that having our students write their metaphors for mathematics in a classroom setting has affected us deeply, has changed the way we interact with individual students, has made us more conscious of the beliefs that students bring to our classrooms, and has made us more direct in our conversations about the way we see mathematics and the way we would like our students to think about it. We urge you to share your experience and ask your own students to write their metaphors for mathematics. The actual protocol we use is presented on the following page.

Before you begin, however, we would like you think about two important questions.

Why Collect Students' Mathematical Metaphors?

You may want them to help you individually respond to each student. After gathering metaphors for the first time Helen wrote, "Being able to match a metaphor to a face, I could fine-tune my support for each student." If this is your goal, you still need to decide if you want to return them to your students or not.

You may want to use the metaphors to set the stage for a discussion of ways to view mathematics. In this case you will want to return them so students can talk about what they wrote. In our experience some students like to read theirs, others like to talk about theirs, and others will remain silent. You may want to write supportive, understanding comments or questions on them, or you may want to return them as you received them. We never grade them. This writing represents their spontaneous, honest views to the questions asked, but it was not written to be evaluated. You may want this discussion to take place in small groups that the students will work in for a period of time. Sharing in this way could help the groups to get acquainted.

You may want to use the metaphors as a gauge of the students' changing perceptions of mathematics. In this case you will want to gather them early and late in the term, semester or academic year. In this case you will not want to discuss them when they are written. We have, however, returned both metaphors to the students late in the year and asked them to write about the similarities and differences that they see in their two metaphors. Students see their changing views and are quite articulate in describing them.

Who Will Read the Students' Metaphors ?

Students need to know who will read what they write *before* they write. Thus, Our introduction of the writing activity goes something like this:

Today we want you to think about mathematics and write about it. We will guide your thinking We want each of you to write what you **really think**, rather than what you think we might believe or what you think we may want to hear. We ask each of you to remain silent during the writing activity so all have a chance to think and write for themselves. When we have finished you may share with your friends if you like. We would like you to hand in what you have written at the end of the period so that we can read it. (We will return your writing to you at our next class meeting).

NOTE: By the way, we never use the word "metaphor" with our classes. For some the word has no meaning; for others the meaning is technical. We don't want the word to inhibit our students' ability to think freely on paper.

Protocol for Gathering Mathematical Metaphors

Developed by Dorothy Buerk, Ithaca College, and
Helen Gibson, Trumansburg Central School
May 1988

- In the classroom setting each student writes his or her own response to each of the following orally given directions. We also write the directions on the board after they are given verbally.
- Pretend that you have to describe mathematics to someone. List all the words or phrases you can think of that you could use. (Wait 5-7 minutes.)
- Imagine yourself doing mathematics either in or out of school. What does doing mathematics feel like for you? List all the words or phrases you would use to describe what doing mathematics feels like to you. (Wait 5-7 minutes.)
- Think about things that mathematics is like for you. List all the things or objects that mathematics is like for you. List each object you choose on a separate line on your paper. (Wait for a few minutes.)
- Let me suggest some categories that you might want to think about to extend your list of things that mathematics is most like for you. (Wait after each suggestion.)

  If math was weather, what kind of weather would it be?
  If math was a food, what food would it be?
  If math was a food, how would you eat it?
  If math was something you can do in your free time,
  what free-time activity would it be?
  If math was a way to travel, what means of travel would it be?
  If math was a color, what color would it be?
  If math was a way to communicate, what way would it be?
  If math was an animal, what animal would it be?
  If math was a building, what kind of building would it be?
  If math was a plant, what kind of plant would it be?
  If math was music, what kind of music would it be?

- Now go back over your list of things that mathematics is like for you and comment briefly on why each thing is like math for you. (Wait.)
- Read over your list of words that describe mathematics. (Wait.)
- Read over your list of feelings you have when doing mathematics. (Wait.)
- Now go down your list of things that mathematics is like for you saying: "Math is _____" and stating each word. (Wait.)
- Choose the object on your list that fits mathematics best for you.
- Think about the ways that mathematics and this object are alike for you.

- Write a paragraph beginning, "For me math is most like a(n) _____." In that paragraph discuss the ways in which your object and mathematics are similar. (For classes where paragraph writing may be difficult we ask students to list all the ways in which their object of choice and mathematics are alike for them.)

Metaphors for Math

What I wanted to do first was to look a little bit at your homework assignment, the metaphors you collected or devised for mathematics, and think about what you would say if you listed some adjectives and feelings and listed some objects that math was most like for you.

- What's a word that comes to mind for you? Cold. Fun. Exciting. Satisfying. Complete. Design. Intruding. Exact.
- Let's get some feelings that you associate with doing mathematics. Wonderful. Controlled. Constrained. I feel like I'm falling off a cliff sometimes. It's free falling.
- In a sense, this falling off a cliff might be...the metaphor you write, for me, might be "math is most like the free fall". The other thing is when you read what they say, "Falling off a cliff," you say, "Oh, oh, that's bad" or "Wait a minute. If it's that sense of free fall, maybe it isn't," which is one reason why students need to develop some metaphors.
- Some other words to describe math?Encompassing. Involved. Challenging. Frustrating. Relaxing.
- What kind of objects did you have on your objects list? Not necessarily the one you chose to develop, but...Pencil. Eraser. Lots of paper.
- One thing that I want to do is think about the words we use and the words students use and then what we can do to try to bring those closer together. That's where the challenge is. The next challenge.
- Are there other words that you use? Potato chips. Eating them...because you can't get enough. Lobster. Of course, the students will say brussels sprouts.

First let us get another list. Words you'd use to describe mathematics that are from students, either yours or the ones that I've given you. What words do students use to describe mathematics?Unfulfilling. Beautiful. Frustrating. Wondrous. Law. Ridiculous. Boring. Obnoxious. Evil.

What feelings do they associate, and again, we've already got some of the collection of feelings, but what kinds of feelings do they include? Concentration. Torture. Emotional. Binding. Wow! Silence, but intense silence. Mind boggling. Meditation. Help!

What Objects do the Students Choose?

Jaw breaker. Oak trees. Tunafish. Brussels Sprouts! Spicy tomato sauce. A large, tall wall. Jail. A magic, supernatural entity. Sunshine. Skyscraper. Blindfold. Puzzle. Traveling backwards, upside down, dragged underwater by a motorboat from New York to London.

You do get some clear messages about where they're coming from. Any comments when you look at these pairs of lists? Usually they're much more positive than negative, but theirs are quite mixed. Their balance isn't bad, but the negatives are pretty clear.

Better Results When a Class Writes about Math

In the writing seminar, I get longer, more developed metaphors because they know it's a writing class and they assume more is expected. I find the classes of stronger math students tend to develop their ideas more fully. Wait time is really important. We can say, "What's the list of words you would use to describe mathematics?" And they put down three or four, put the pencil down and we wait three or four or five minutes, and they begin to get another idea. "Okay, if nothing else is going to happen, maybe I'll get an idea and maybe I'll add to the list." So that the lists get longer the more patient you are and the more encouraging you are.

Student Metaphors

What I'm going to do with you is share some of the metaphors that students have written and give you an idea of what we're trying to find out about these. Let me first read to you one metaphor:

> To me, math is like a used car that you get for a good price. Sometimes it runs smoothly, but on certain days things go wrong. It's frustrating like a car can be when it won't go right. You have to sweat, yell, curse, and sometimes pay a price to get the car going. But once it goes, everything's great. With math, things don't always work out right. I don't know how many times I've screamed, pulled my hair out, trying to fix a math problem, but when I finally figure it out, I feel fantastic, like I've accomplished something. Sometimes you break down in a car or during a math problem, but if you work at it, you'll get to where you're going.

A theme that comes through in this one is that math is both easy and hard. And it's clear that you see both sides of it. In terms of the feelings, they're angry, they're frustrated, but they're also happy, satisfied; they feel like persevering. Again, you get both sides of it. But the overall theme of this

metaphor is that these people are in power. These people are in charge. These people are doing it. It's their struggle, and they'll yell and scream and pound if they get really excited, but they really feel in charge of what they're doing. And there are a lot of metaphors where the powerlessness is really clear. The questions is, "Who's in charge?" and for whatever they say, if they feel empowered, you've got a lot going for you.

We looked at 100 metaphors from high school students, and they were all juniors and seniors in math courses. About twelve or fourteen students talked about disliking math; that math was uninteresting; that math was challenging; that math was defined by relationship; that there was some sense of a wholeness; that math takes a long time. There was just one theme that seemed to occur more in these metaphors from males than females. It's that notion of useful. It's important. That was a theme in 31 of the metaphors, but 22 of those were from males, only 9 were females.

There are a number of themes that the females tended to use more than the males did: that math is easy, understandable; it's interesting, fascinating; it's not bounded; it's difficult—and difficult was in 43 of the metaphors; it's complicated; it's a collection of different methods. While there were some boys and girls that chose those, it was more a theme that girls picked out than the boys. "I feel happy, satisfied"—that feeling of accomplishment; being exhilarated; feeling like persevering. Those were themes that came more from female metaphors than from male metaphors. And almost half the female metaphors talked about being excited and exhilarated. It may have been that balance of "I'm excited when I get it right." or "It might be awful." but it was a theme that was in the metaphors.

Student Metaphors that Inform Our Teaching

One student wrote,

> For me, math is like an earthquake. Seeing as if an earthquake was to hit, even just a tremor, it could knock down and ruin a lot of things. Just like in math, if you make one error in a problem—even a small one—it can ruin or tear down all of your work.

And in fact, the teacher had decided that she really wanted the students to understand that being careful and accurate was important so she gave no partial credit on quizzes. When she read about three metaphors like this, she said, "That's not a good idea. I understand philosophically why I'm doing that, but the conception of mathematics that they're getting is not the conception of mathematics that I want. I've got to think about that in a different way. I don't want them to believe it's either 100% correct or it's 100% wrong. That's really a problem."

Helen Gibson, the high school teacher who worked with me on these (we did this activity with all of her classes), sat down and read the metaphors of her classes one night, and went back to all of her classes the next day and answered their questions, got them to talk about what they thought the experience was like, got into some—in one class, a whole period—discussion about their concerns about mathematics. The fact that somebody cared amazed one class, "It was the first time that we've had a discussion like this." For some students, especially the students with negative attitudes, they are relieved that you know.

There are times when you get metaphors that scream out that this is a student you need to be conscious of. One student wrote, "Math is like quicksand. The only way I can be safe is for someone to pull me out." But think about math as quicksand. I'm sure our strategy is to try to gradually get the student to be active, to take more of a role, to try things. In quicksand, anything you do makes it worse. It reminds me of a student who wrote, "It's better not to try than to try and be wrong."

I think the most disturbing metaphor I've ever gotten is this:

> For me math is like an abusive parent. If you do not stay on guard, your parent will strike you when you least expect it. It's not rational and it's difficult to understand why things are the way they are. Parents and math are supposed to help you get through life, not screw you up and hinder you. I hide from the parent, yet it always seems to find me. And that sense that math is out to get me, no matter what I do.

Again, you need to be very conscious of this student and how to provide support.And one thing Helen has said is that she looks at her students and she knows how to respond to the students in different ways because she knows them by their metaphors.

And let me read one that I fell in love with a long time ago and just can't let go of.

> For me, math is like a good hiding place that I can escape to, think in and be comforted by. It's in another world away from interruptions. It's filled with rare objects and known objects that I'm drawn to and mesmerized by. I stare at each one of these objects for long periods of time. I have all the time in the world in this place, trying to unravel its mysteries and trying to find out what they really consist of. Some are easy to discover; others are very difficult, almost impossible. When I've had no luck with these more difficult objects, I wander back into the cave, the center of this secret place. In this cave are many different crystals and many beautiful, but well known, rocks and gems. Here lie all the basic truths. The foundations upon which everything is based. And then I go back out again, renewed with strength.

I want to move on to the next piece of your assignment. We've thought a little bit about your views of math and students' views of math. What I want you to begin to think about—and hopefully you've already begun—is the third question on the list. "Which of my beliefs, feelings and values would I most like my female students to embrace?"

What we Hope our Students Believe and Value

Offerings from the Group:

- Well, we thought confidence in their ability, especially for girls, because they tend to be a little more hesitant.
- They need to understand that math is a growing body of knowledge and not something which is static. We both felt that the sense of joy is something that is really important.
- Yes. The mystery, the excitement, the wow...that kind of magic that we would like them to feel.
- I'd like them to know what math can do, how it's constructed and have sort of a fond sense of relationship to it so that if there is ever a time when they want to go back to math, they can do that.
- And the sense of exactness...we want them to understand the procedures that work and do lead somewhere. I'm learning accepted procedures, but I'm learning them because I have some sense of ownership and I have some understanding of them.
- Usefulness of mathematics, not seen as a bunch of symbols. I'd like for my students to do a math course more as a problem-solving course.
- I want my students to understand the interdisciplinary aspects and pressures and complications of mathematics and not giving up on an answer.

The notion of usefulness...again, what does usefulness mean and what does it mean to students? I had one group of writing seminar students who kept talking about and were excited that "This is useful." What do they mean by 'useful'? They don't mean that I can go around and count pine cones all the time. So I asked them on the mid-term, because you can always find out all this kind of stuff by asking them to write about something in a writing oriented course or ask them to write a journal. "What is useful? What would make math useful to you? Are there any examples of topics in this course that you feel useful?" And what they seemed to be saying was that useful, for them, and these were both males and females, was connected with: "I understand it." "I can do it." "It makes sense to me." "That somehow I have ownership of it." And it wasn't the more traditional: "I can apply it to some

real word problem I'm going to have," but "It's useful if I understand it." And that's a whole different meaning of useful then we think about it. I think when you listen to the words they use, we still need to go back and figure out what they mean. What do they mean by abstract? What do they mean by useful? What do they mean by a whole variety of these things? Because what they're talking about often is very different from what we may think, especially "useful". I just kept reading what they were saying, and saying that somehow they're not using that word the way...I know that what they're thinking isn't the way I use that word, but then what is it that?

Skills Aren't Gifts; They Can be Learned

What I'd like to do is think a little more—and you've been doing this already—about some strategies that you use, and would suggest, to help students begin to move closer to having these beliefs, having these values, having these kinds of feelings.

I've talked to teachers who have just totally reconstructed their rooms to do cooperative learning. For some people the seats are always going to be in fours and "that's the way we're going to function", but for other people that's too radical a change. And the people who have done that with cooperative learning have found that there's a group of teachers who have formed a network in our area. If they want to do as much as possible by cooperative learning, they design some neat activities to teach this topic and then ask, "What can I do with this and this". They've begun to get together and have formed a network. "This works." "I want to introduce polar coordinates. What might I do?" They are sharing activities because the text materials for learning all sorts of things by cooperative education aren't really available.

And some people's thing is not other people's thing. And we can't do all cooperative learning and have them always writing or doing creative problem solving and deal with the curriculum. We have to figure out what we can do with our plans, with our personalities, the demands that are happening, the expectations of our students and parents and headmaster/mistress.

What I'd like you to think about is what is some strategy that you tried. What is some question you have about where I might go next. And try to share those. So try to come up with strategies and some particular concerns and we'll share those.

The session ends with group discussion still going on... ❖

Questions for Teachers Who Want to Enhance Student Learning

Dorothy Buerk, Ithaca College

- What are my female students thinking and believing about mathematics? If I don't know, how can I begin to find out?
- What do I believe, feel, and value about mathematics?
- Which of my beliefs, feelings, and values would I most like my female students to embrace?
- How can I help my female students see and experience mathematics in this new way?
- What resistance do I expect from my female students? What can I do to learn to accept my female students' resistance to change? What can I do to lessen their resistance?
- What will hinder me from changing my female students' experience with mathematics? (Possibly local or state curriculum guidelines and demands, or the expectations of students, administrators, and other faculty members, etc.) Which of these things can I control? What can I do about them? Which of these are beyond my control? How will they impede my progress?
- How can I get started? What is most important to me as a first step? What pace should I set? How much can I do and how quickly?
- What support system do I need to implement my plan? How can I develop that support network for myself?

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Cornelia Brunner

Research Scientist,
The Center for Children and Technology
Bank Street College of Education

CORNELIA BRUNNER'S PH.D. from Columbia is in the field of Developmental Psychology and the Psychology of Art. She is on the faculty of the Bank Street College of Education where she teaches in the Computers in Education Program. She has designed educational computer software for middle and high school students and worked as a consultant to the Girls Club on Operation S.M.A.R.T. a National Science Foundation project to support math, science, and technology education in informal settings. Ms. Brunner has been a research consultant to PBS for math science programming on *Sesame Street* and *The Electric Company.* ◆

Margaret Honey

Research Scientist,
The Center for Children and Technology
Bank Street College of Education

MARGARET HONEY HOLDS A PH.D. in Developmental Psychology from Columbia University. She has researched and published on women in technology, use of laptops for education, teachers'understanding of technology integration, and empowering women and minorities. She has consulted with the Children's Television Workshop and taught courses on the psychological development of women.

MS. HONEY AND MS. BRUNNER have presented reports to the American Educational Research Association, the International Congress on Women, the National Education Computing Conference, and the National Women's Studies Association. ❖

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Gender and Technology

~ *Exploring the question of technological imagination* ~

Cornelia Brunner & Margaret Honey

IN TODAY'S WORKSHOP, we plan to explore the question of technological imagination by having participants engage in a hands-on activity that involves describing an imaginary device. In the course of the workshop we will look at participants' "imaginings" and we will explore three issues in relation to the activity. First, we will address issues having to do with technology and society—what is the culture that surrounds technology, and what are the implications of this culture for gender issues? Second, how do we think about the makers of technological objects—who are engineerings, designers, etc., and what are the gender laden assumptions we bring to this question? Third, how do we view technology itself—do we see technological objects as facilitators of human actions or do we see them as god-like and all powerful?

EXCERPTS FROM THE DISCUSSION FOLLOW.

Origins of the Study

HONEY: I'm Margaret Honey and this is Cornelia Brunner. We both work at the Center for Children and Technology, which is a research and development group that's part of Bank Street College of Education. The kind of work that we do at the center has to do with educational technology. These days we do a lot of work helping schools and school districts to think about ways in which they integrate technology into their curriculum. But that's not why we're here.

The reason we're here is that several years ago we began a research study that had to do with questions of gender and technology. A group of us at the Center for Children and Technology — a group of women at the Center — started this project and it was really very much started by our own thinking about this, our point of view. We literally sat down and began to talk about our relationships to technology. One of the issues that came up for us was that even though we use technology constantly in our work—Cornelia never goes anywhere now without her little portable computer—we noticed, and

it took a slightly different form in all of us, a reticence and a reluctance to get deeply involved and deeply engaged in technology. We treated it as though, "Yes, it's great stuff but it doesn't really matter. We don't want to get too close. We don't want to get on too intimate terms with it." In other words, there was some way in which technology, the presence of technology, put us into some sort of conflict. It raised some sort of issue that prevented our deep engagement with it.

So what we proposed to do, and what we were funded to do, was a series of studies which looked into these questions. And we decided to take a very different approach from what had been done in the past. We all know that girls and women don't use technology as much as do boys and men. We know all about the deficit model that's been well and very importantly documented in the literature. What we wanted to do was to take a look at the ways in which women and girls understood and interpreted technologies, ways in which they went about establishing relationships to different kinds of technological objects. So we wanted to look at processes of interpretation, patterns of making meeting, patterns of appropriation, all of those kinds of issues.

So what we proposed to do was to study a group of women who were technological experts and then study groups of adolescent girls who were technological novices. What our funder ended up asking us to do was to turn this into a comparison study and include a sample of men and sample of boys as well, which proved to be very useful in the long run.

BRUNNER: The erstwhile Girls' Clubs, now known as Girls, Inc. has a program called Operation S.M.A.R.T., which is designed to introduce girls, in an informal afternoon setting, to math, science, and technology. They made something which they call a research kit, which allows the girls to do certain kinds of evaluational assessment research on themselves and their peers.

The object of this little game or activity is twofold. One of them—and the one that is most obvious for the Girls' Clubs, but not necessarily so important here—is to have girls think about how they associate gender differences with technology. In other words, going on the assumption that most girls will think that machines are invented, designed, used, etc. by men, and that women play only a very particular and relatively small and helpful role in all that development, one part of the activity is designed to help them talk about that in some way. But another part of the activity, which is really more interesting to us and what we use it for, is designed to feed into an ongoing sort of research focus, that has to do with finding different routes into technology for girls and women.

The study helps us think about that, and what we do is create programs, curricula, etc. that allow girls to do two things. One of them is to enter the world of thinking about technology, fantasizing about technology, without the bottleneck of having to know a whole lot of math and science before they feel entitled to do so. And we think of that as developing their technological imagination. This is connected to all kinds of other research. It has to do with

considering different ways of thinking about this stuff, all legitimate; rather than just one way or the just the mainstream way.

Developing a Technological Imagination

This is a very small example. When you explain technology based on sports metaphors, you tend to leave out anyone who doesn't think that way. So we're always interested in looking at other ways of conceptualizing, interpreting, thinking about, and imagining these kinds of objects. And the major purpose of this little activity is to find out how girls, who really have relatively little concrete knowledge about technological objects, might imagine them, and we give them permission to do so. And then we have an interest in analyzing what we get from here to develop a different set of introductory concepts that we all want to legitimate. We want to say, "Well, if you think about technology that way, that's cool, this is what follows," rather than "If you're using this particular metaphor for technology, you're wrong." You have to switch metaphors, so that's essentially the key to the context in which we developed this little activity.

Students are shown pictures of parts of an imaginary machine. They are asked to decide what each part does and what the overall purpose of the machine will be. They are also shown photographs of a group of men and women from many age groups and ethnic backgrounds and are asked to choose the designer of the machine from among the choices of pictures. The activity booklet distributed to the students included drawings of the different parts of the make-believe machinery, faces, patterns, and areas to write. The students then invented a machine, describing its form and function, composed a team of inventors, and elaborated upon its anticipated uses and benefits. The instructions from the Editor to the "Science Report Team" for the "April Fools Issue" of the *Hampton Hills Gazette* read as follows:

> I want you to write a really scientific sounding story about the TECHNOLOGY OF THE FUTURE show that's in New York this week. I sent everything you need: some pictures of a great new machine and of people at the show. Pretend the machine won first prize and the people are the team who invented it.
>
> Write me a story in which you tell:
> a) what the machine is called
> b) what it is for and how it works
> c) who invented it—give people made-up names
>
> Tell as much as you can—and make it seem real. I want scientists, engineers and other experts to be fooled. I want a lot of red faces when we print in the next day's paper that it was all made up!
> Have fun!

Thinking Past the Stereotypes

The other thing that can come up is that if children are going to discuss the kinds of jobs that they imagine people have, there are certain stereotypes that now crop up: the mad inventor, the management-wife-mother who sort of takes care of everyone while mad inventors go off and do their numbers. But the way we think about it is not a matter of just knowledge; it's a matter of actually having students, especially girls, talk about the conflicts that arise when you imagine yourself seriously doing this kind of work. We all know that if you're 11 or 12 years old, you may think it's possible for you to do this, but if you're 13 or 14, deep down you know you're going to have to choose between sex and science, or sex and technology. I mean, you know that you can't really do both. You can't really be a mad inventor and work with machines like that and still be a management-wife-mother sort.

So one of the ways in which we like to use these kinds of techniques is to start a different kind of conversation about what it means to take on those roles. We know that our minds can do it, but does our world actually permit us to do it? Girls' Clubs just recently did a very interesting ethnographic study of how the clubs do after school science at different sites all over the country. They found that the regional subculture where these clubs are determines how science is taught. So in the South, there's a great deal of emphasis on respecting others and being very traditional; not questioning authority, not doing anything that messes up or breaks the rules. In the Northeast, there's a lot of individual striving, and even competitiveness, as long as you don't do anything that breaks the rules. In other words, as long as you have permission from adults to do it, you're allowed to explore, but it has to happen within a schedule. In the Northwest, there's even more freedom. Kids can really explore and figure things out and do things on their own, provided they have no real conflict. In other words, as soon as you do something that creates real conflict, the grown-ups come in and tell you to stop it. Regional cultural differences determine how we actually teach math and science technology more than anything else.

Examining the Conflicts

And we, as the people teaching it, have to become aware of the fact that the cultural baggage that we bring to this is a very real contradiction with what we generally think and speak of as really good, innovative math and science technology, all about questioning things, questioning authority, being messy, not really knowing what you're doing, but doing it anyhow. All that kind of stuff really is in serious conflict with our much deeper notions, especially of femininity. Any kind of admission that these conflicts go on and if you have to keep resolving them, is on the one hand very useful, on the other very hard to do because most of you who are teaching math and science, if

you're women particularly, have resolved some of this for yourself. And it's hard to open it up again and act like it's an ongoing issue. It's much easier to say, "See, it can be done" and leave it at that. So it can be complicated and painful, and we have to be much more aware of how delicate and fragile our own dance is. In other words, our own way of constantly re-resolving this conflict a little bit this way, a little bit that way, a way of compensating for having spent too much time doing something that is interesting and absorbing so that you don't notice that your children aren't trying to get your attention because you're really into what you're doing and then you compensate for that. The more sensitive we become to these conflicts as adults, the more we can actually encourage girls to look at conflict and to find ways to cope.

One of the things that we found in the study is that, of course, all the women who are tech experts are constantly confronted with these kinds of conflicts in their world. Even when they're working in a place where all values and ethics and all kinds of assumptions about the nature of the world are quite shared among the genders, women find themselves in a much more conflict ridden position in this world.

Forms of Conflict

HONEY: Also, conflict resolution takes very different forms, at least in our sample of women. For example, the women architects we interviewed, either in their work or as part of their professional training, felt it was absolutely essential to develop a deep understanding and knowledge of structures which, for those of you who know anything about architecture, is not essential because you can call in the engineer. But these women didn't feel themselves to be legitimate; they didn't feel that they could compete with their male colleagues, be as respected as professionals, until they had that deep understanding. I'm only talking about two women here, but the interesting thing is that both of them told us these long stories about their particular career histories, and both of them had worked for very prestigious architectural and engineering firms, and at a certain point both, in some way, opted out. One of them, at the time we interviewed her, had decided to go into the business of designing jewelry boxes, and the other one had become a teacher, a post-secondary teacher of undergraduates who were going to major in architecture and engineering. But in some way they both took ten steps back from what they had been attempting to do and ended up in something we might call a more feminine space or place, and they were deeply satisfied with what they were doing at this point in their lives. But the story they told us about how they had got from here to there was filled with conflict.

We were interviewing a woman who was a computer scientist and did a lot of programming work. One of the series of questions we had explored was

what their work meant to them, what really turned them on and what was deeply important to them about their work. She had this great line, a kind of disclaimer, where she said, "This technology is really not as important to me as something like my needlepoint or my deep, deep love of murder mysteries." There was some way in which her utterly, totally competent, and clearly respected from a professional point of view, involvement with technology — it goes back to what I was saying at the beginning— couldn't allow her to be deeply engaged and connect with that; it couldn't be as meaningful to her as pursuits that we think of as typically more feminine. We're not alone with that. In some sense, Cornelia and I can get deeply involved with technology only to the extent that it has something to do with education, another much more typically feminine pursuit. But to get passionate about a machine as an object in and of itself isn't permissible.

BRUNNER: In almost all the interviews, women keep saying that they know how to deal with this machine, whatever it is, their instrument of work, totally, but it's clearly not what they're interested in. They're really interested in helping others. They're really interested in making the machine more accessible to people. They're really interested in this and that...it's always in a context of a certain kind of service, and the machine is useful in that context. Men talk about the machine.

Metaphors

The metaphors women have for these technological objects tend to be more that it is a fellow creature of some kind. A fellow creature that needs care and feeding like all other creatures, and it does things for us if we treat it well. The thing that it does for us, more often than not, is to connect us to others, either by expression or communication.

The basic metaphor that men tend to have about this object is that it is, in fact, a kind of magic wand. That it not only expands their power and efficiency or speed or something like that, but that also allows them, in a certain sense, to bypass some of the genuine activities of communication and expression and creation. By a mind zap, enhanced by technology, they have a direct, extraordinary influence on the physical universe. So you have a thought, and cities are created, lakes, mountains. That is a genuinely attractive fantasy.

HONEY: People can't buy into it. There are a lot of people in the population who can't buy into that; that look at it as magic. And they can't join the party.

BRUNNER: One way to talk about it — back to the conflict — is to say that their wet dream is my nightmare. The whole idea that you could have this extraordinary power of the universe for having a thought scares me. I don't want to touch anything that pretends or purports to be able to do that. That leaves me out. I don't want any part of it. A legitimate way of looking at this as a means of communication and expression might invite me in, but

somebody has to do the work of explaining to me that it really can be looked at that way, not just that it can be used that way. But my way of understanding it, thinking about it, talking about it, my personal set of metaphors for it, are just as legitimate as the ones that come from weaponry or stuff like that. And it takes much more work than we think because the common language that we share about these objects tends to come from that other world.

What I want you to do now, when you share your machine, is to think about which of these universes they belong to. Who wants to start sharing their machine?

Machines People Make

This is off-the-wall. Josie has conceptualized a machine which is called Josie's Jobit. She envisioned a machine because she was always losing her marbles. She was a marble shooting champion and she was always losing her marbles, so if you look at this machine, there is one area where you can put in a large sphere and it goes up and becomes increasingly smaller and then bounces down and gets on this little thingamajig and it goes up and comes down the chute. So this is the way she would always have her marbles.

So it helps her keep track of her marbles?

Um-hum. She can make more. She's a marble shooting champion and she needs them. She goes out and gets the big spheres and they become the small spheres.

BRUNNER: Okay. So we have a marble maker.

I don't have down who designed it, but it's name is Samantha Doolittle. And it's called a Countadiddaly. It's supposed to take numbers of base 10 and turn them into numbers of base 8. The idea is that this scope down here at the bottom, you punch the number in there, 10, I guess you punch it like this...it goes in here and turns this wheel around, and this wheel goes around 8 times and it counts — knocks off one of these little balls up here at the top — and when the little ball here at the top goes over the dial here, it turns and the ball falls on the floor and dial turns and registers something, and then this little business right here has a pen on it, and makes a mark on the sheet of paper.

BRUNNER: Both of you are doing something that, when we use it as a diagnostic tool, children don't do, you both talk about the particular functions of parts of the machine. And kids don't do that even though you're supposed to tell what each one does. What you will get most of the time is kids talking about the different parts as if they were just different magic boxes for different things, and no relation between form and function.

HONEY: Who'll go next?

I have one. Mine is a used gum baller, and its use is to dispose of used chewing gum. Part A was used to stop and start the machine. And in Part B

it flattens the gum out into a sheet, and Part C it screws it up into a little flattened piece, and Part D rolls it up into a ball and tosses it into the wastebasket. It's not recyclable.

BRUNNER: These are all very useful. They do something that needs to be done. There could be another way to do it. This would just be nicer. Who else?

FURTHER GROUP EXAMPLES

BRUNNER: How many have we got? Foods? Chocolate and coffee.

Gumballs, potatoes, water.

BRUNNER: So half are food makers. If you were kids, there would be more homework helpers. So far the math one is the only one that has that quality in it.

What other kinds of things did kids turn out?

HONEY: A wish granter, which I think your machine is a much more sophisticated version than that, but that's real common. Actually, some kids we gave this to thought of a bottle cap screwer. But again, they don't think about the connections between different parts; it as though the parts could sometimes belong to completely different objects. So they're not connected into any sort of whole. Homework helpers are big ones.

BRUNNER: There are three sort of basic categories that girls make. They make helpers. It facilitates something that you do already in the kitchen, cleaning, etc., and makes it a clever facilitator. Or, things like homework helpers which are beyond clever; they sort of do it for you. And they make friends. They make extension creatures of some kind that they can talk to, watch TV with. Make TVS that feed you. And really, I'm not exaggerating, by and large, boys make weapons and vehicles, especially vehicles. And we can't be sure that that isn't a sample problem because since we tend to be women, it's possible that if we really did this more systematically, where the boys felt comfortable and didn't have any pressures on being politically correct whatsoever, we might get a better set of weapons. We'd still get vehicles once in a while; vehicles that take them to outer space, take them somewhere faster, deeper, better, or something like that. They draw them with great precision and, if we asked, they labeled the parts so we'd have a sense of what the parts do and how they relate to each other. And most of the parts enhance the power of the thing in some way.

When we analyze in detail what kids actually do, the overlap between girls and boys is much greater than the overlap between men and women.

HONEY: The kids' data is interesting because overall it looks more like the adult women's data. What we saw in the adult data with the unusual object was that the men would come at it with an operating metaphor, a guiding metaphor, and that they would use that as an organizing principle and they would, in relationship to the metaphor, systematically interpret the different parts of this object and explain how the functions worked. And the metaphor was almost always a lathe.

BRUNNER: It was a Mexican orange peeler. You put the orange in there and you turn it and screw it.

HONEY: It was very odd looking, but it did look sort of like a lathe. So they use that as their guiding interpretative object. The women approached the object much more globally. They would tinker all of the different parts, pulling here and pulling there, making speculations about this and that. And then maybe they would guess about what the object was. But their explorations with it were much more overall and across the board.

And the kids tended to be more like the adult women. And there weren't, in our sample of adolescent kids, clearly pronounced gender differences as there were in the adult sample.

BRUNNER: And one way to interpret that is to say that men tend to be more interested by the object itself. They tend to look at the object, and women tend to look through the object to its function. And so men know more and are more interested, and are more sophisticated in their language about describing the object itself.

HONEY: And that's true in general. In relationship to our particular sample, because the adults were technology experts, we know from our interviews with these people that almost all of the men in our samples had a lot of hands-on tinkering experience. They had a lot of familiarity with different kinds of objects and mechanical objects. So those metaphors were already in their heads and they brought them to bear on this stuff, whereas the boys in our sample, we don't really know actually how much hands-on tinkering they had done, but that would be something to check out, to research, because other studies suggest that that makes a big difference.

Does conflict explain why, for example, Korean students do so much better in math and science? In Korea, math is very, very important.

BRUNNER: And apparently, not in major contradiction, you can be a good Chinese girl, let's say, and a math whiz. I don't think you can be a good WASP girl and a math whiz.

What do you mean by that?

Well, I think if you're going to be a math whiz, almost by definition, you can't really be a good girl. You can't be totally altruistic, totally outer directed, totally psychological, totally all the things you're supposed to be and be a math whiz. And have a date on Saturday night and be a math whiz.

You just can't do it. From our research also, there's a real question about whether or not you know you can have a date on Saturday night.

Perceptions of Careers

The other thing that we found that is related to this, is that when men and women tell the story of their career, they tell it very differently. Now this is not surprising, but it is just staggeringly true each time. That when you hear a man talk about his career, he will talk about it as if he made a decision at a certain point in his life and then he pursued it, then he did this and he did that, and this opportunity arose and he took it. You hear a woman tell what could be the identical story, she found herself over here and then somebody said...and then she said, "Maybe I'll..." And "then somehow..." And "then next thing you know..." Same career, same story. The idea being that this pursuit of a goal is in itself extremely suspect. Thinking of our lives as something where we made up our minds to do something, now I know these guys telling these stories didn't make up their minds any more than the women did. In fact, the women have to make up their minds more to get there. But they are able to see it as something that has a certain self direction.

And we're not allowed to know that about our own careers because if we do, you know what you really are. I mean, if you have vision, if you have a goal and you can see it...

HONEY: This is a little bit of anecdotal data from our interviews. The women who we interviewed who did go to all girls' schools talked about those experiences as being incredibly informative, very important for giving them the confidence to pursue mathematics, or to pursue work in the sciences, or whatever they were doing. It didn't make a difference in how they told the story of their careers. We had very high-powered, very professional women running companies. They still said, "Oh, I just ended up here." And if you think about yourself, how many times have you told your story. I've done it a hundred times — until I became aware of it — in exactly the same manner..."I don't know how I ended up here." "I needed a job."

BRUNNER: We have an anecdotal story about a coed school. A girls' school was coeducating a couple of years ago; they were going to merge with a boys' school. The headmistress of the school decided that she needed to introduce technology. She offered a technology course; first year 23 girls — successful. Next year 28 girls took it — successful. Third year, final year she was giving it, 40 girls took it — very successful course. It was given again the year after that, the first year the two schools had merged. Forty kids took it again, three of them were girls. There's data like that from all kinds of places.

The conflict — to get back to it — that we think has to be dealt with in a way that "positive messages" don't do the trick, is that if you look at the image that we have of scientists, mathematicians and inventors, let's say, to make technology real for kids, that image is what?

Very bright. Eccentric. Yes, Eccentric. Dedicated. Devoted. Nothing else, blinders on. Spends years, hours in the lab. Antisocial. That's what it takes to become an Edison, right? That's what it takes to become a Nobel prize winning scientist or mathematician. Who wants to do that?

So that even though we tell girls they're smart enough and they're good enough, we don't address that conflict underneath that you have to be a geek to actually succeed with this stuff. And the way we tell our career stories, we're still talking the same way. "It just happened so I'm not responsible for having chosen this particular path that got me here; I'm just an exception." In my case it works. I manage to maintain my femininity.

HONEY: The interesting thing about our adult women in the study is that what we heard in our interviews is a struggle to find a different way of interpreting their relationships to their work, a different way of making meaning out of it. We hear a lot of what we call "internal talk," about building bridges, about making connections. Several of the women who were into hard-core computer science, doing work in artificial intelligence, ended up later in their careers getting much more interested in things like user interface, how do you design a program so that others can connect with it, and make use of it so that it can facilitate things. We hear that language again and again and again.

You don't hear that talk from the men at all. The men, when you ask them what turns them on about their work or what makes it meaningful, will give you detailed descriptions of a specific problem that they encounter. And it's often a problem that sort of takes you deeper into the object so it's figuring out a complicated piece of code where you have to pinpoint the exact moment that's causing your program to crash, or something like that. But those are the kinds of descriptions that we get. So they sort of take you into the technology, into the object, where the women, as Cornelia was saying, sort of see through, they treat it much more transparently. You see their involvement with it as a process of building bridges.

And I think that in teaching girls about this stuff, we're trying to get them interested in it. It's very important for us to be cognizant of that and to encourage that perspective as legitimate, as an alternative to bigger, better, faster stuff because it is important.

BRUNNER: And to be good at that, it takes the kind of skills that we have much less conflict about having. For example, taking apart a car motor or any motor, taking apart any object has an onus for girls that creating and building does not.

HONEY: A design tool that Cornelia invented is a draw tool. It runs on the Mac. So kids can just start drawing anything they want. And then with the help of the instructor and with certain tools that are built into the program, they can get much more systematic in their thinking about these machines. They can label their different parts. They can begin to think about the relationship between the parts. And then they can take the next step, if they

want, and animate. But the bottom line with all of this is that it creates a space in which girls are allowed to think of themselves as designers, which they've never done before. They've never really been given the opportunity to say, "Okay, invent something and go ahead and draw it." And it doesn't matter how wild or fantastic or crazy the idea is. Whatever your idea is is worth doing and important, and it's a real sort of legitimating experience for them. And they don't have to get through, what Cornelia was describing as the traditional bottleneck of math and science. But hopefully, in the best of circumstances, it can work as a bridge out to getting them interested in the physics of design, the mathematics of design.

BRUNNER: And it's okay for girls to be designers, engineers. But again, we try to start them out with: What are you imagining? What does it do? Why would we need it? What makes it start? What makes it go? So there are certain questions that lead you into thinking about your machine. Sometimes we have them start out with a machine that really annoys the hell out of them; fix it, imagine making a more congenial machine. So there's different ways, depending on where the kids are. But just allow girls to imagine themselves as the inventors.

TECHNOLOGICAL LANGUAGE

The language, the very language that we use to talk about and think about technology, can be extremely off putting. My favorite example is a very simple one. If you look at a computer, you can describe the interactions that you have with the computer as either issuing a command or picking from a menu. If you think about the difference between those two terms which describe exactly the same behavior and exactly the same aspect of technology, there's an extraordinary difference in what it says about the nature of the technological object. If you have to command it, not only do you have images of army men dying because you issued the wrong command, but also you have to be in charge; you have to be controller; you have to know what you're doing.

Whereas, picking from a menu you make the wrong choice, big deal. You pick another one later. It feeds you. It gives you something. It lets you do what you want. It's a completely different image of what the object is that you're interacting with. And I don't think we can overestimate the amount of difference, whether we're conscious of it or not.

BRUNNER: At some level, as this technology gets integrated into our lives it becomes more consumer technology; it gets demystified and girls also feel more comfortable. But it happens very gradually.

And one of the things that I think we have to do more of is just to be cognizant of the kinds of explanatory metaphors that we use without ever legitimating them or speaking them out loud when we try to interpret the technology we use every day. Refrigerators, televisions, etc. If you didn't

know as much as you do, you have some way of explaining this to yourself and very often it will be a kind of a creature metaphor. Very often you'll attribute to the object motives, feelings and reactions that you know perfectly well it doesn't really have. When we hear those very same explanatory ideas clothed in what we consider the appropriate language, if you look at it, it's just as irrational, but it's okay because it comes from a world that we have legitimated as being one through which you can talk about this stuff, which is very different from saying that we should only deal with kitchen technologies because that's what girls like. Any technology can be thought of in ways that are more like the way we quite spontaneously think of things.

Session Ends with a Group Discussion. ❖

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Instruction With Cooperative Small Groups

~ *Does it really influence individual learning?* ~

Carolyn Hopp

THIS WORKSHOP WILL FOCUS on what the research says in answer to that question and to another. How can we, as educators, design the group context to promote beneficial interactions and discourage detrimental interactions among group members? Come with your questions, your answers, your concerns about using small cooperative groups in your mathematics or science classes. The emphasis here will be sharing insights and methods. Participants are asked to bring, if possible, a task or activity that has been used successfully in a cooperative group setting.

> REQUIRED READING: *Guided, Cooperative Learning and Individual Knowledge Acquisition* by Ann L. Brown and Annemarie Palincsar, from *Knowing, Learning and Instruction: Essays in Honor of Robert Glaser*. Hillsdale, N.J., Lawrence Erlbaum Associates.

EXCERPTS FROM THE DISCUSSION FOLLOW.

Well, I found this interesting. I didn't bring this newspaper because the headline says, "Wisconsin Kids Near Top in Math". I knew this press release was coming because of my work with the Department of Public Instruction. I was wondering how Midwest newspapers would handle it.

You look a little further, to the subheading, "But national experts say no state teaches students well enough." I read this article with interest just to see if there would be any reference to gender or cooperative groups. Well, on page 8A...and I don't know if it's Ina or V.S. Mullis, an executive with Educational Testing Service, said, "Teachers should use technology and encourage students to work in small groups and do projects in math." The big focus today is on cooperative small groups as an instructional method.

In *Newsweek*, look what made the cover, not the math scores, but *Secret Warriors in the Gulf*. Anyway, I read with interest a 2-page article about the NAEP results that have been released for eighth graders in 37 participating states. "Fewer than a third of the students surveyed regularly spent time in small group work and the vast majority had never done mathematics projects or written a report on math. Again, don't you get the feeling that, yes, we should be doing this, but how? And what's the curriculum we should be using? Well, that's my whole focus. I looked for gender differences, but that's not the focus of the talk today, although we do have a little reference here. "The results showed no difference between the performance levels of girls and boys in grades 4 and 8. However, in the 12th grade, boys did better than girls, suggesting that cultural pressures don't kick in until high school."

How many of you are in all-girl schools? I don't know how these figures translate to your setting. Anyway, I kept reading and trying to make this talk relevant. I have just joined the Research on Women in Education Special Interest Group of the American Education for Research Association, and I recently received a publication of theirs, *The Women's Educational Equity Act Publishing Center* publication. There's an article in here about Pat Campbell that I think Judith Jacobs referred to the other night. I thought, well, let's see what Pat says, because I know her. She writes about the need for intervention programs to get young girls to continue in math and science. She says, "How do we reach large numbers of teachers to do this?... many of the characteristics of effective programs—more hands-on and fun work and less individual competition can and should become part of math and science classes."

My feeling is that you science people are ahead of us mathematics teachers because you're already using cooperative small groups, right? You're working with projects and lab experiments, and I'm sure you have some questions about concerns. Hopefully we'll have time to talk about them today.

What's All the Fuss About?

I'm here today because of a tremendous interest in teaching, but also in cooperative learning, and I decided that what I would probably do is rename this workshop. I kind of like this as a title "What's all the fuss about cooperative learning?"—because there certainly is a fuss that's being made right now. I think the problem is that we're in an age of competition. Bob Slavin, a researcher out of Hopkins with whom I've become more familiar in the last couple of years, says that the age of cooperation is approaching and that he thinks its time has come.

Recently a friend of mine gave me a wonderful book. Alphie Kohn is the author. If you are a believer in competition, you won't be after you have read even the first chapter in this book, *No Contest: The Case Against Competition*. Mr. Kohn lives in a cooperative household in Cambridge, Massachu-

setts. He says that he sees competition as our number one obsession as Americans. We grade kids. The woman I've been working with on my research study said, "Carolyn, if I could do away with grades, I'd do it; but then what do we do instead?"

So I wanted to set the stage and tell you that I'm interested in cooperative groups, not only for these reasons, but also because I consider myself a victim of a competitive society. I'm 47 years old. I'm now in the third year of a doctoral program. When I started graduate school two years ago, I had taught for almost twenty years, nine and a half in the public schools, middle school through high school, and nine at the University of Wisconsin - Eau Claire. In all those years, I had not learned to work collaboratively. When I got to University of Wisconsin, Madison three years ago, people started handing me articles and I thought, "What is this? People are sharing with me?"

And now I'm in graduate school and I'm supposed to be talking about issues. And I'm supposed to be agreeing or disagreeing with people, and you wouldn't believe how difficult it's been for me. I've had to speak up. I never learned to agree or disagree with classmates. We never talked about mathematics in math classes except with the teacher. We did explain processes though and strategies at the board, and I can remember a very good geometry teacher of mine who had us do proofs at the board without notes. That was scary stuff, but I think it helped me because I had to teach my peers.

In the research, I read these studies—I don't know if you're familiar with what the research says—but supposedly and there are many inconsistencies in the studies—the research results say that learning in cooperative groups is certainly as effective as learning in a traditional setting. I thought, okay, I'm going to be the skeptic here and pursue this a little further. The kinds of tests that are often used in these studies are guess what? Multiple choice. Retention of basic facts. Very little emphasis on problem-solving. And I said to myself, "Okay, what does that mean?" It's a field that is new in terms of really looking at achievement and problem-solving in mathematics.

So I became interested in the kinds of tests, and then I started reading further, and I found that studies of peer interactions in cooperative small groups very rarely mentioned the activities the students were working on. They might say, "Students spent 20 minutes each period working on homework or worksheets," and since many of these studies were in the late 70s, I thought, "What kind of activities are we really talking about here?" Worksheets...is this more basic facts than problem-solving? So my research interest is and has been to look at how student interactions are different when kids work on different kinds of activities. I've just spent time tape recording and video recording students working in small groups in an 8th grade classroom while they worked on different tasks, and later you're going to become research subjects, and I'm going to involve you with one of those and maybe more than one, if we have time.

Probably the most important point is that cooperative groups have to be effective if we're going to use them. We can't just throw kids together and then never summarize, never process what went on. Again, I keep referring to the two teachers I worked with, they were so wonderful at the middle school level. They would do just that and tell me, "Carolyn, we can't do another task tomorrow. I have to process this with the kids." So the teacher would use the next day to summarize, and they would talk about the students' strategies. And she'd call them back together. I don't think anybody is saying you do it *all* cooperatively. At some point, I, as a student, would want some quiet time where I could think by myself.

I wish what we are doing here could be a whole day. I would like to take you through a task that I used in my study and have you react to it and discuss whether you think this is a good activity, and if so, why. My problem and my frustration is with tasks. Teachers have said to me, "Carolyn, I don't know what kind of activities I should use." So when I talk to you about how you design the group context to benefit individual learning, I consider the task to be critical. Therefore, the focus today is on effective tasks for instruction with cooperative groups.

I don't know if you got the little note asking you to bring a task that you have used. I envision that we could all walk away from here with a task from each other person; a task that you consider to be effective.

To share resources that I have discovered, I have a packet of materials where the focus is on mathematics. There's some wonderful work done with graphing. Maybe I could share these the last half hour because I have been fortunate to work with the Department of Public Instruction in the last year and to become familiar with curriculum materials from Australia and England that I've got to tell you about. They're incredible. And they can be used with traditional textbooks as a supplement. They're grades 7 through 12, and some are even elementary. You've got to see the material.

Without further ado, how do you want to get into groups today? How many do we have? Do we have seventeen? Do you mind if we count off? Groups of three. Your first objective is to take five minutes and briefly talk about why you think cooperative groups are valuable. We're going to make this brief because I want to get to activities, so we will summarize as quickly as possible.

Can we say things against it?

You can definitely say things against it—reasons for not using it. Sure.

Group Work

Let's just go quickly. I just want to jot down what you see as reasons for using cooperative groups. Hopefully you've talked about reasons for not doing so, too, but I want to dwell on the positive a little bit more than the negative today since I think this is something you might want to consider. I'm

not saying do it; you have to buy into it or you're not going to do it anyway.

Can we mention some of the questions we have as problems so maybe we could address those and decide that they're not problems?

Yes. Why don't we just start with this group here. Shall we say that these are pros?

Focuses information on students understanding processes of problem-solving by explaining and verbalizing.

Students take possession of their knowledge and build self-confidence. And this is hand in hand. I think everything works together.

So by taking ownership, this implies self-confidence?

Right. It builds self-confidence.

Wonderful. Anything else?

Yes, two more things. Allow students to discover math and science.

Empower students.

And if that isn't the word of the 90s. I like it. Did you have questions or concerns in this group?

Well, I did. I'm concerned about the misuse idea. I think it is a concern. I think it takes an enormous amount of preparation and, as you said, the tasks are extremely important. And so the idea of just forming groups and letting them work is not what we're doing.

One other thing is having sources, materials suitable for this kind of activity.

Absolutely. I think that once you see a few that are effective, you can start creating your own. In fact, you science teachers have been doing this a lot, I think, and hats off to you.

Improve social skills and communication

Tolerance.

Better understanding through verbalizing or explaining.

Using these groups does not allow the target students to control the class.

Not enough time.

And it's my problem today. There's no doubt about it. I'd love to have you hear a student protocol today, but I don't think there's going to be time.

How to grade.

Yes. By the way, here one of your best resources is other people who have been using cooperative groups; talk to each other. I think that we're finally trying, in our schools, to get teachers to be less isolated, more open. And we may not get answers to all these today. That's a bigger frustration for me, but at least we can talk about issues and I'm glad we're raising them. Anything else you want to point out?

We had a couple that were different. Each person gets a lot of attention.

So, attention for individuals.

Now we're talking about groups that ideally function as groups here, aren't we? And it takes training to get the kids to that point. It doesn't just happen overnight. Anything you want to add though?

Concerns

Over on the concerns. Selling it to parents is a big concern.

There are always a few parents who come and say, "Why aren't you doing it because I read about it in..."

And others are saying, "Why aren't you teaching my child individually?" Truthfully, there I think that it would be very helpful to point to one or two research articles where they talk about the value that you've got right here: verbalizing, teaching your peers, in fact, has been shown to be related to individual learning.

What would be useful is a one-page insightful statement that incorporates the research that we send out on back-to-school night.

Ideally, on back-to-school night, when they're just sitting in the classroom, give them a task that they will cooperate on.

I was going to say why not get them involved in an activity that may not require high level mathematics, but is a task that is real world, in which they end up talking to each other. I wish I had run this article off. Both the ones I ran off for you are more theoretical. I apologize. I've been working on this too long. They're really good articles, but this article, I think you will find it helpful. I highly recommend this. This woman is a dynamite woman, Nell Noddings. She is really a philosopher, believe it or not. She talks a lot like Gilligan does about connected ways of learning for women. It's an article in the *Elementary School Journal*, entitled *Theoretical and Practical Concerns About Small Groups in Mathematics*. It's a wonderful summary of how to group and some of the issues associated with it, and the purposes of using groups. It's very, very good.

The question has been raised about what to do with an unwilling, uncooperative group member. I had one in my study, by the way, a young man who considered himself entirely too bright to work with a group. He would not do it. He worked totally by himself, and I couldn't intervene.

You know what I did with that? I let him work alone. This was in an elementary situation and soon the solitary student noticed that everybody else in the class was having a lot of fun. And at the end of it, he actually rejoined. It took him a while to sort of get over that.

You're not going to be able to make them. Well, I shouldn't say that. There are people who would say that you assign a role to them and at least then they're doing something and not tuning everybody else out. I would try that.

This group? Anything you want to add? It's always terrible being the last people to report.

Somebody mentioned something about grading them. One thing that we agreed on was that whether you work in a cooperative group or not, there should be something that each student turns in that they're responsible for, at least doing the writing, rather than just sitting there. This does not allow the lazy students to sit because they must do something.

Individual Accountability

Individual accountability is a very, very important key component and that's what Bob Slavin really emphasizes along with the motivational component. And that's an excellent point. One thing that I tried very successfully is a group hand-in where the responsibility alternates. For example, I would maybe have three very high level mathematical tasks or problem sets and they would come together as a group. And one person would be in charge of writing those solutions up the first time. I would count it the equivalent of perhaps a quiz grade. And the grade was a group grade for that particular assignment, but the responsibility of writing it up rotated. It worked beautifully. Now I don't know that middle school kids could handle that, but I think they could. I would try it anyway. But I know that high school students can. They could decide who would be the first one to write it up; that might be more the confident student who says, "Well, I'm willing to." But it's not that that person has to do the thinking; they have to pull together, do their thinking on strategies and come up with a group product, which is really collaboration.

Cultural Influences

One thing that we talked about was kids who culturally would not participate.

Open up kids who won't culturally do so. Is that what you're saying?

Yes.

What cultures are you talking about?

Asian. They don't speak in their classrooms unless they are spoken to.

Of course. They're not used to speaking up; they're used to being spoken to. Now I know what you're going to say. The Asian students often score highest on tests.

We have to question the test; what type of testing are we doing?

I want to know more about that because that's made me think now. But we could get into a whole discussion about that.

There are so many cultural, family time dedications. There are so many other factors involved with the Asian culture that we can't possibly compare to that.

Can you see how complex this is, though? When I started trying to do research I thought, I'll never be able to do this because there are so many variables, but I still refuse to give up.

Individual Cognition

All right. Now, my whole crunch here is—and I know I said in the talk—we want to talk about what it is that happens in small groups that influences

individual learning. That is, individual cognition. My advisor was a skeptic at the beginning. She said, "Carolyn, I believe with Piaget in what's called self-directed learning, that learning takes place within the individual. And you're really going to have to work to convince me that cooperative groups are going to make learning occur." I think I've convinced her.

There's a Russian named Vygotsky who wrote the book *Mind and Society*. That's on the reference sheet, too. Vygotsky talks about the social genesis of understanding. In fact, that by interacting with others and experiencing those processes we internalize them and we learn by ourselves. These two theories, Piaget's and Vygotsky's, are now being brought together.

Let's talk about this so we can get a handle on it. On the right hand side, skip to the third reference for now, the white sheet called Classroom Materials and References. I would triple star Elizabeth Cohen. She has done wonderful work with minority students in looking at status differences. I highly recommend her book. She redefines intelligence so that we don't always talk about the I.Q. Did we talk about different kinds of abilities? I love the chapter that she has in this book called *Multiple Abilities*. "Gee, maybe I can visualize. Maybe I'm very good at reading. Maybe I'm very good at thinking logically." She lists a number of abilities that we need to consider. We tend to just pick the student with the highest I.Q. and think that's the person who's most talented. It's a wonderful book.

Get It Together, math problems for groups, I've heard teachers say good things about. There are task activities in it, for grades 4 through 12 where individuals are given clues to a problem, and they have to share them with the group. You know that kind of thing? I was going to do that with you today, but it wasn't my focus. There are also Equals Publications out of Berkeley. If you're not familiar with Equals and you care about gender issues, it's really worth getting in contact with these people. *Get It Together*, math problems for grades 4 through 12. If you're interested in training kids to work together, that's a dynamite book with activities you might try, getting them to learn to talk up and listen to each other. It's a very good resource, with some high level mathematics, too, for high school kids, I've been told.

The green reference sheet is for those of you who want to know more about the research. These are references that I have found to be wonderfully helpful. I probably don't have Vygotsky on here simply because it was more theoretical, but I can certainly give you the name of the book. It was written in 1978 and it's called *Mind In Society*. I warn you it is quite high level reading with quite a bit of jargon. You might want to just go to the library and look at, if I recall, Chapter 6. It helped me to understand theoretically what he's talking about when he refers to "the social genesis of understanding." That is, we learn by interacting with each other and then we internalize. I was so happy to see some of you say that. I wrote ownership and you wrote internalizing. These are just wonderful comments that you have made.

Before we go any further though, let's take a look at an activity I used in

my study. I would like you to work this activity as a group, however you want to do it. Tell me if you think it's a good task to get kids to talk with each other and to learn mathematics.

I was so happy to see the social skills come up here. They're very important, but the emphasis today is on learning, and learning mathematics and mathematical thinking and processes. So you decide as a group how you would approach this task; how you think students would interact with it. Be aware of how you *feel* as you do it and what you say as you complete it.

Group Activity

I'm going to interrupt just one second. I'm really interested in your graph on page 2. I'm going to pick a group at random to present their graph.

Let's just take a minute or two just for fun to look at what some people got. Now think about this activity and whether it's a desirable one. Think of ways you could modify it. Think of what you could do with a class discussion with this. I could have each group put up their graphs, and I would not give them the correct answer and say, "Well, what do you think?"

We started at 0 because he was parked and we got up to 30,but we didn't do that right away. We had originally just started right at 30. And then he probably slowed down to about 15 because he was on his way to school and there might have been kids around when he was going around the corners, and we gave him about a quarter of a mile to turn, then back up, another corner and then he slowed down again on the way back to school so he could drop Peter off.

Now, do you agree? Disagree? Was your graph different? You can see where the discussion could go, and I can't get into that right now. How many of you agree fairly well with what they've got up there? What would you do?

That second curve, we'd end up going slower because it's a sharp curve.

So what are you saying? That this curve then should go down lower?

Yes, maybe to 10 miles.

I have another question for you calculus teachers. Will these be curves or will these be like this?

Curves.

Are those graphs different?

Yes.

Can you imagine the discussion you can get into? I wonder if it should be a series of segments or should it be curves? You can get kids ready for precalculus concepts. Now I know there are prior questions and concerns, but I want to point a couple of things out. First, if you look at the activity, notice the directions. In order to do my study, I had to have the same task structure for all the tasks. They would not ask the teacher to help, but they would talk to each other first, and then only when they had a question that nobody could answer would they talk to the teacher. Those were the

directions on each of the five tasks that I studied. And this was considered a non-routine task, but think about how you could take some of these and apply them to higher level courses. How we could modify this task so that we could use it beyond the 9th grade level?

I like it for physics.

Let's talk about the mathematics in it. Look at what they're doing. They're taking the story and they're translating it to the graph on page 2, right. On page 1 they're going from a graphical representation to a verbal. These are all skills that the National Council of Teachers of Mathematics *Curriculum and Evaluation Standards* recommend. Did you make this up?

No, I wish I had. I could get rich on it maybe. But I'll tell you where it's from. Take out your folder. We're going to do things a little differently here than I thought in the beginning. Now these are not inexpensive materials and I know that your schools' budgets are tight. On the right hand side of the goldenrod sheet are the materials designed for assessment at the University of Nottingham. These are for secondary schools and I ordered them for the Department of Public Instruction not knowing the equivalency rates for pounds. A bill came — are you ready, nobody fired me — both of these together came to a total of $242, not inexpensive. I don't know if your Department of Public Instruction in your state could get these for you through a resource center. I'm sure that in the state of Connecticut, Steve Leinwand has access. The man is working in assessment and doing incredible things, and if he doesn't have these I'll be really surprised.

When you said you got them all, you got the software video package?

Yes, and I haven't even looked at the software or the video, but you talk about dynamite material that teaches kids to not only tell stories about graphs, but also to look at graphs and interpret them. This can be tied into secondary content beautifully.

Constructive Controversy

And if you take nothing else away from here today, tell yourself—and I believe I'm going to find this in my study—that tasks can encourage disagreement as well as constructive controversy. Are tasks that influence constructive controversy very effective? I have research to support that. And so what I'm concerned about is that you leave here thinking that you should be going with non-routine tasks if you're trying to teach concepts in math and science, tasks that will get kids to disagree, tasks that use intuition. Researchers call it constructive controversy. They're wonderful, and there are more samples here.

If you're interested in articles on constructive controversy, that green reference list has an article by the Johnsons entitled *Conflict in the Classroom.*

That task, as I said, was developed for individual assessment, and yet I

used it in groups and got incredible interactions from the kids. And, in fact, do we have that protocol? I wanted you to hear a tape of a group that I recorded in my pilot study who seemed to disagree all the time anyway, but for this task, in particular, there was some constructive controversy where they would disagree, but then they would defend themselves and have to rethink what they didn't understand.

In this protocol there's a young woman named Barbara. On the fourth page, she says, "You guys really ought to listen to me" because she had the right answer and she was ignored. That's a powerful statement in that protocol. In another one, she says, "Listen to me. Listen to me. *Listen* to me!" That's her plea in a protocol. And if you can get kids to hear somebody in the group saying that, don't you think they might attend to that person the next time? I don't know.

But see if you can get kids to disagree and then constructively resolve those differences, because when somebody disagrees with you, you have to figure out whether you really know what you're talking about.

Other Materials

If you want to see what eighth graders in my pilot study did, just take two minutes to read the protocol and look for disagreements and how they resolve them. That is the focus of my research, dissension episodes, and I'm looking for gender differences, too, in groups of four that have two boys and two girls. I'm going to see who resolves those dissension episodes and whose answer prevails in the end. Is it the girls or is it the boys?

Also, these new materials from Australia are much more reasonable and are described on the blue sheet in the folder.

What grades are these designed for?

This is for grades K through 12. And each book has activities for all. I highly recommend Volume 2 of the Activity Bank. These are materials from Australia, and Doug Clark is one of the co-authors. He is now a grad student at Madison and I can talk to him any time I want to! He's here on a two-year leave of absence. Volume 2 contains an entire chapter on cooperative activities. And if you look at that entire book, you'll find ways of adapting activities to your classroom, K through 12. They're beautiful. I was going to bring copies with me, but we flew and there's a limit to what I can bring with me, but I wish I had.

What does Volume 1 have?

Volume 1 contains activities which are also K through 12. They're organized around themes. There's a chapter on visualization. There's a chapter on getting kids to be aware of what they read and how it looks at gender. I think Volume 2 is more geared toward secondary, but don't quote me on that. I wish I had them with me now. As you can see, they're not prohibitively expensive. And I would want to own these myself. I have not

seen American materials in my work that are as good.

How about computer software?

That I have not seen. If you want to correspond with Doug Clark and simply write to UW-Madison, WI, Center for Education Research, it would get there. Or contact me. Do you have any information on me? I don't know if you know how to get hold of me. Let me give you my home address though because we shift offices often at the University. I really feel so strongly about all of this that I wish there was some way we could network with each other nationally. Maybe that will be a dream come true some day.

Suggestions for Cooperative Group Activities

Now, any middle school people here? Wonderful. Maybe you already know of some activities. Can you tell me of any good things you've discovered with cooperative group activities?

Yes, I do a big unit on finance. They make checkbooks and they make up checks and then we have ten folders with three items of each. Some were bills, some were salary checks, and they have to decide what to do with each thing. They'll write checks for the dividends and they discover from listening to someone else that money is coming in. Then when they've done all that, we give them a bank statement and a balance.

Was that something you brought a copy of with you? How many of you have a copy of an activity that you could contribute today?

I have one that's adaptable. It's more for elementary school. Basically I just bring in all the food I happen to have for ten people and they either have a budget or they don't have a budget. And they have to plan a meal and do everything from tablecloth on.

I love it.

It brings in nutrition and all sorts of other stuff. One group decided to cater the whole thing.

I have to point out another thing. If I don't mention this today, I'll never forgive myself. On the classroom materials and resource sheet, the white sheet, there are materials developed by Addison Wesley. Actually they're developed by Michigan State University. I had a dream that if I didn't care where I moved to, I'd go to Michigan State because they have researchers who are teachers. They're linking research and teaching. If you go down to Glenda Lappin, about 4/5ths of the way down the page, *The Middle Grades Mathematics Projects* is a set of six, I believe, modules that are designed for use for grades 5 through 8. They have the most challenging mathematics that I've ever seen at the middle school level, using manipulatives in a creative way. They're incredible. And these are American materials that I think will knock your socks off. And you've got to hear the Addison Wesley group at your next conference. Or talk to a middle school teacher. See if you can get them on approval. I cannot recommend them highly enough, and I'm not

doing that as a VPI rep because you can't do that. I might do this as an educator because they include precalculus concepts, just like what you're talking about. They're not cheap, but maybe there are ways to get around that. Both teachers whom I worked with in my pilot study and in my research study used *Middle Grades Mathematics Projects* materials. They work cooperatively, of course, with people at the lower grades because the materials are designed to be used at all levels. And there's some review built in: maximizing volume, minimizing volume, surface area, those kinds of things. If you can get a hold of an Addison Wesley book, they're in there. And the modules are maybe $70 to $80 total, but then, of course, you've got the manipulative expense. Nothing's cheap here.

Our time is up, darn it. In the left hand side — I'll just take a minute — to point out four other tasks that I think you may enjoy. And I can't emphasize enough, of course, connecting this to the content we're doing. We're not going to teach these as isolated activities. The vending machine, this is also from the Shell materials, the English materials. It's a lot of fun. I've seen teachers, including myself, struggle with that one for a few minutes. Stepping Stones is also from that particular series.

Which series?

The Shell series which were the Nottingham materials on that goldenrod sheet. These two, Add the Numbers One to One Hundred, and Fermi problems. These are from the Australian materials. And Doug, the author, told me the Fermi problems are incredible for cooperative group activity at the start. The question is: How many piano tuners are there in Australia? And the kids look at you and say, "Come on. Who cares?" It's a chance for them to pool their knowledge. If you read the Brown and Palincsar article for today, you read about the role of support in a group. And I'm sorry that I didn't summarize all that, but it's there. Doug told me the Fermi problems are wonderful for any age. You just make up a problem which is open ended, and they have a wild time with it. And then Add the Numbers One to One Hundred uses computers in a beautiful way — technology in this one.

You've been great. I really enjoyed it, and I'm sorry I got so rushed at the end. I knew I would. But thanks a lot. Share with each other. That's the whole thing. I hope you will try it if you haven't tried groups before. It's incredible what you can hear when you start listening to individual students. ❖

Claudia Henrion

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CLAUDIA HENRION IS ON THE MATH FACULTY at Middlebury College, where she teaches courses in Calculus, Computers, Logic, and the History of Mathematics. Her undergraduate degree from Stanford is in Philosophy and Mathematics. Her Ph.D. in Logic is from Dartmouth, where she is currently working during her sabbatical year. In her teaching, Ms. Henrion has explored such areas as Gender, Science and Computer Science, Minority Studies, and the Math Anxious Student. Alternative approaches to knowing and learning mathematics and to the history of mathematics have been themes of recent lectures. She has published her research in the *Humanistic Mathematics Newsletters, Journal of Symbolic Logic,* and the *Proceedings of the National Conference on Women in Mathematics and the Sciences.* Her work in progress is *Dialogues with Women in Mathematics: Their Lives and Their Work.* ❖

Placing Mathematics in a Cultural and Historical Context

Claudia Henrion

ONE PARTICULARLY EFFECTIVE WAY to motivate mathematics is to try to find connections between mathematics and students' own lives. This involves exploring the human dimension of mathematics: reestablishing connections between mathematics and the people who do math, as well as looking at the relationship between math and the culture in which it is practiced. This is particularly important for women and girls who often feel more alienated from mathematics, and who see mathematics and human concerns as mutually exclusive. In this workshop we will consider ways of presenting mathematics as a human activity and include hands-on activities to illustrate some of the major points.

EXCERPTS FROM THE DISCUSSION FOLLOW.

I'm truly delighted by the mixture of different people we have here, from different places within this country and beyond, and different backgrounds. I really encourage you to share the things that you've learned, the things that you've done in your classes, as we go on with this workshop. At specific times, I'll really set aside time to do that, but if anything comes up that you want to share at the moment and you just can't wait, by all means, chirp in.

My Autobiography

I want to begin by expanding a little on the biography I was introduced with. I'm choosing to begin in an autobiographical mode partly because I find that beginning with biographies or autobiographies is an extremely useful technique in the classroom, especially with students and even in a mathematics classroom. It's something that I did in the course I'll be talking about shortly, but I think you can do it in any math classroom. I begin by having students write their mathematical autobiographies, what they experience.

For many of them, it's the first time they've sat back and reflected on their experiences in mathematics. I think it's also a way to engage students immediately in making math more relevant to their lives.

I want to tell you a little bit more about how I came to mathematics and how I came to be here today, to set this workshop in a larger setting. As a high school student, I was actually at an all-girls' high school in New York, and being in that environment, I never questioned being a woman in mathematics. It seemed very natural at the time. It was something that I liked to do and many of my peers liked to do. And I learned early on that cooperation didn't need to be in conflict with competition, that you could even have a healthy kind of competition in the classroom and it wasn't a bad word. There are healthy ways of doing it and not so healthy ways of doing it. And I learned about working with other students and that seemed like a natural forum for learning mathematics.

Those lessons were in sharp contrast to my experience when I started my undergraduate work at Stanford, which was a much less supportive type of environment. At the time I think it's changed some since then — there was an outward disdain for reaching out to the students. It was clear that the priorities there were about research and not about teaching, and I watched many talented students just leave mathematics because of that. So it taught me early on about the importance of the atmosphere within a classroom in sustaining students' interest. In fact, my way of compensating for the lack of human contact was to double major in math and philosophy, and in philosophy there was a lot more interaction and stimulation. It turned out to be very useful in my future interests, though at the time the two were quite separate, my math and my philosophy. I really didn't see any connection between them.

It wasn't until graduate school, when I started working in logic and subtheory that I began to see the overlap between math and philosophy. But even then it was minimal because in mathematics graduate school we were really focusing in a specialized area and not thinking about the larger question of sort of fundamental questions: What do we mean by mathematics? Why do we do it? How does it fit into a larger picture? For all of these reasons the further along I got in graduate school, the more I found I needed to answer some of these fundamental questions. Part of that was stimulated by beginning to wonder if there were any women in mathematics, because in my last year of graduate school, and many of you might have experienced the same thing, I had never had a female teacher, professor; I had never heard of any female mathematicians. And so I began to wonder if there were any, and if there were, how did they do it? How did they combine their personal lives with a life of mathematics?

Where Are the Women in Mathematics?

So I started trying to track down any literature I could find on women in mathematics. Now, many of you might have already found yourselves looking for such material, and it's quite minimal. There's not a lot of material, and most of it has come out in the last ten years. One of the things I'm going to do today is share material with you that I've tracked down over the years and that I think might be useful for you in your different settings.

I began in trying to read anything I could find about women in mathematics and then doing a little bit of research on the history of women in this country in mathematics and, in particular, I studied one woman, Helen Merrill, who was here at Wellesley very early on. She was one of the first graduates and later became a professor here for most of her life. And so to try to learn: What was that like to be a woman doing mathematics at that time? How did they work that out with their lives? And it was a fascinating process, and it was the first time where research sort of hit me at a gut level, where it really mattered to me in a personal way. And that was an exciting experience, too.

That interest in historical women in mathematics led to a piece published in a book called *Women in Mathematics, a Bio-Bibliographic Source Book*. This is a book filled with women mathematicians, mostly in this country, up through about 1950 or 1960, there are a few from other countries, but the focus is pretty much American mathematicians and it's edited by Grinstein and Campbell, and I believe it's Greenwood Press. So if you ever have students writing papers about women in mathematics, for example, it's a good source to send them to. The pieces are very short, but then they have references to other material.

Starting with the history of women in math was very interesting to me, but I wanted to know how that related to contemporary women in mathematics. Are things so different now than they were before? And that led to my current research on a book of contemporary women mathematicians, in which I'm doing intensive interviews with women around the country, that hopefully will be out in about a year or so.

The reason I bring this up is because when you begin to look at these topics, you're quickly led to even larger questions about not only what is the relationship between women and their lives in mathematics, but what are the larger cultural issues that affect individual mathematicians and mathematics itself? In other words, how is the mathematics that we do and create influenced by social values, belief systems, social, cultural, religious, political factors? So I really became interested in that interplay between mathematics and society, and how people fit into that picture.

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These interests took up a variety of forms, but the one I'm going to talk about now is a course that I was actually able to teach as a freshman seminar at Middlebury, on the history of mathematics. My interest in doing this course was to try and find a way to make the history of mathematics more interesting than the standard image you would conjure up when you think of the history of mathematics. It's a very dry subject with a lot of memorization of names, dates, and people. There were three parts to the course. The title of it was *An Alternative Approach to the History of Mathematics*, and the first part of the course involved looking at the lives of individual people in mathematics. I'm going to talk more about that in just a second. Let me outline the three parts first. In the second part of the course, not only did we look at individual people, but we looked at the larger cultural issues, so we looked at how mathematics developed in different cultures and in different time periods. The third part of the course looked at just one branch of mathematics and traced the history of it. I focused on geometry.

Now what this did was really develop three perspectives on the history of mathematics. So immediately students began to think that there isn't just one way to approach the history of mathematics, but indeed many different ways to do it. And that depending on which approach you take, you see very different things in the history. With the first part of the course, we focused predominantly on the big name people that you hear about in mathematics. We went back as far as we could and still have written material on people such as the Greeks and later people, but we also tried to work and make sure we covered different women in mathematics throughout time where we could get information about that.

At the end of this part of the course, we made sure we brought it up to contemporary people in mathematics and there's a good book in the bibliography called *Mathematical People*. It's number 3. I don't know if you've seen that or not, but it talks about the lives of modern mathematicians. Making that connection to modern day people is very useful for students because they still think of mathematics as something way out in the distant past, and some of the people are quite colorful; some started out as magicians or jugglers and then eventually found their ways into lives as professional mathematicians.

So we talked not only about what their lives were like, but how their lives were influenced by social factors. So, for example, in reading about Sonja Kovalevskaya, it is important to understand Russian culture. She was part of this early neolith movement in Russia in which many of the people believed in emancipating the serfs and in liberating the people, and the way they believed in doing that was to not only liberate women but do it through science. They felt that science was the avenue to empowerment.

There was a lot of encouragement at the time for women to pursue science,

and Sonja Kovalevskaya was very much a part of this movement. So, in some sense, she had a lot of cultural support to pursue a life of mathematics. On the other hand, the institutions were closed to her. They wouldn't let her in, Russian institutions. She had to go abroad to study mathematics. And the only way she could get abroad was if she entered into a fictitious marriage because women were not allowed to travel alone. So a lot of the circumstances of her life were influenced by these factors. She ended getting a Ph.D. at a German university, and she wrote not one, but three, different theses in order to be sure that she would get a Ph.D. She went on to win some very prestigious awards and got a position in Stockholm.

Very quickly, in studying the lives of these individual mathematicians, you begin to look at these other cultural factors which is a natural transition into the second part of the course. In the second part of the course we looked at mathematics in different cultures, but we didn't just start with the Greeks. Most history books just start with the Greeks as if math suddenly appeared with the Greeks and, of course, it was long before that and so it's important to see how the Greeks were influenced by early Egyptian, Babylonian, African mathematics. So we began to read materials about that and it's mind boggling to hear that the Pythagorean Theorem, for example, existed well over a thousand years before Pythagoras ever lived. It was certainly in Chinese cultures and it was in other cultures.

So you begin to think about why we call it the Pyhthagorean Theorem; who gets to be named on different discoveries and things like that. Well, it turns out that one of the reasons we call it the Pythagorean Theorem, or Pythagoras is credited with it is because he was the first, so it's claimed, to come up with an abstract proof of the theorem. Before that, a lot of people could generate Pythagorean triples but they couldn't really do this general proof of the theorem. Well, then you get into some interesting questions about what constitutes a proof. When do you really know when you have an abstract proof of something? And those are very deep questions about what we count as a proof. And students begin to think about this notion of proof that we take as bedrock and solid, and it changes over time, and different cultures have different ideas of what's an adequate proof. They might try and use that line on you in an exam or something. So something I think is important to try to convey to the students is that the concept of mathematics and proof evolve over time in different cultures. So those are some of the things that we're looking at in this second part of the course.

Finally, in the third part of the course, we look at the history of geometry. That worked in very nicely with the first two parts of the course because we started with pre-Greek geometry and then saw how the Greeks formalized it in axioms and then how geometry evolved so that now we think not just in terms of one geometry but, indeed, many geometries. That there's not only Euclidian geometry, which takes place on a plane, but you can have non-Euclidian geometries, which take place on, for example, a sphere. And that

also is somewhat mind boggling the first time you learn about it; that it's no longer clear that there is one mathematics, one geometry, so then how do we decide which geometry we work with, or which mathematics is true?

So the idea behind this course was to really get students to begin thinking about: What is mathematics? What do we mean by that? What do we mean by history? And how do these concepts change over time? And how are they related to the people and cultures in which they are developed? A little aside. When we're talking about the topic of what do we mean by history, I have the students do the following exercise—it was shortly after the massacre in Tiananmen Square and I had them each write for a few minutes about what had happened there. But I began by asking how many students had heard of Tiananmen Square, and everybody but one person raised their hand and said, "Yes." So I told that one person to just hold off for a minute and I'll be clear in a minute. After the students wrote for ten minutes, I had them read what they had written, and they each had quite different perspectives on what had happened here. The idea behind this exercise was to convey how our different perspectives of history can be quite different and 2000 years from now one of their documents might be the only surviving document of what happened. So we had to be very careful in thinking about what documents do we have to work with and what that teaches us about the past.

Well, as it turns out, this one student who said he hadn't heard of Tiananmen said, "Oh, of course, I heard about it. I just couldn't figure out what it had to do with mathematics." He thought it was something like the Pythagorean square or rectangle or something. That's a great example of context, you know. So there were a lot of things that we did in this class, and what I've described to you is pretty much the content of the class. But equally important were a few relevant points about the methods used in this course. It was a discussion course and it was very nice to be able to have a discussion in the context of the math class. That's a luxury that we don't often have, and it's great when driven entirely by questions. In the beginning, I provided questions, but as the class went on, the students were in charge and thinking about their own questions. I'm amazed how little we do that, especially in colleges, and I think, to a certain extent, in high schools. We don't spend enough time having students generate their own questions. If you think about it, most of the real learning that you do comes when you have thought about the question. So that's something that we really tried to emphasize in this course.

There was also a fair amount of writing in the course. We did both an informal and a formal kind of writing. The informal writing was with the use of journals. The formal writing was in some short papers. They did a paper for the first two sections of the course, one in each, and gave presentations on the papers. But I think the most rewarding assignment for the course was the final project. And it was the following: they were to do a history of mathematics timeline and they could do that in any way they wanted, using

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any medium they wanted. So, for example, one student did a creative sort of storybook for his younger brother on the history of mathematics. Two students got together and did a video of the history of mathematics. Several did fairly elaborate graphic posters of the history of mathematics.

The Mathematics Quilt

My favorite, by far, was the project two women students collaborated on. They did a history of mathematics quilt for the course. It was so cleverly done. They did it with construction paper rather than material, and it was a series of different colored and different shaped patches. Each thing was quite symbolic so certain colors represented certain cultures and they wrote information on each patch so they had pieces of information about early Egyptian mathematics on one color and information about early Greek mathematics on another. Different shapes stood for different aspects of history; if it was a square, then it was information about a person. If it was another shape, it was information about the culture. They had in each corner of the quilt a different time period; 17th, 18th, 19th century mathematics. And they did little touches, like they had black patches to represent all the people we don't hear about who have contributed to mathematics. And on the edges of the quilt, they had fringes to represent that mathematics was continuously growing and developing. In this one quilt, they had really embodied much of what had happened in the course in a very creative form. It was a reminder to me of the creative potential of students if you give them rein to express what they've learned. So that was very exciting.

That's a general overview of the course and some of the methods that were used in it. Are there any questions about that? A lot of the materials, in terms of references, are in the bibliography that I handed out, so if you are thinking about using any of this, you can certainly do that.

Questions

- What was the ratio, boys to girls, in the class?
  It was about one third women and the rest were male.
- Were students who majored in mathematics required to take a history of mathematics?
  No, there's virtually no history of math. There was one offered before this class. My class was a freshman seminar so it wasn't necessarily math majors who were taking it. It's not required?
  Not required. It's definitely optional, and that makes a big difference.
- In my senior classes I do start off, usually in the first week or so, asking them to do some kind of their autobiography of math. With varying learning styles, I wanted to be sure and hit the differences so I did the same thing, like you did with your projects. I said, "What I want

you to do is think about the history of your mathematic life. What has happened in your life; you can tell me any way that you want to as long as I understand what happens from mommy saying, 'How old are you?'— two or whatever — all the way up to now. And I find the same thing. It was very exciting. I had pictorials. I had posters. I had acting out. Any way, it was a very, very popular and very successful project, one of those things that was way beyond my imagination. They took it and ran. It was really interesting. And I had some that really disliked mathematics and it came across, if I may say, in a positive way. I mean they were able to express it: their fear, 'I don't like it,' anxieties and so on and so forth. It was interesting.

That's really great. And what that illustrates is that you can really use that at any age. You have a history already, no matter where you get them.

It's personalized. It gets you started in a personal way because you're touching within.

- Has anyone else used that yet in their classes? Something like that? Other things? Some people have taught history of math; I'm wondering if you could share with us at this point any ideas that you have come across that have been useful in conveying the history of math?

  I get my seventh graders to do a short project where we go to the library and I get a whole lot of books and they have to go through and choose—they can do a mathematician, they can do math games and they choose, frequently, to do puzzles and codes and games, and things like that. In order to choose they have to go through a lot and read a lot. They really come up with a lot of things and they'll have a difficult time making choices. Some of the things they do are very interesting. Sometimes it will be historical, sometimes it will just be a book from Gardner.

Group dialogue continues.

Useful References

This is really useful, to exchange what you've found to be useful references. A couple of others I'd like to point out in this list are: Teri Perl's books, #18, I've just written it as *Math Equals*. That is a series of short biographies of women mathematicians plus math activities related to what they did. And the activities are very creatively done. So that's a real must if you haven't seen that. She has another one called *Women, Numbers and Dreams*, which is a younger audience book. I recommend that one, too.

A couple of other books that I'll be talking about more in the workshop, but I'll just point them out now, which have become essential in my library. One is a book called *Africa Counts*, and I'll be reading passages from that.

And a very new book which just came out this year, *Ethnomathematics*, by Marcia Ascher, which I added at the bottom under "other books," #1. Both are extremely rich resources for a multicultural perspective on mathematics.

I'll suggest one just off the top of my head. Each year we give an award for the outstanding student in mathematics at graduation and the award is Eves' two volume book *In Mathematical Circles* where the first volume is quadrants one and two, and the second volume is quadrants three and four, and they're done by degrees. And if you wanted to read something in a class that's applicable one day, you just open the book anywhere a pick a degree, like 95 degrees and then give a short anecdote or something about the history of mathematics. It's just wonderful. Eves has done about six sets of books. This one is *In Mathematical Circles* but there's one, *Mathematical Circles Revisited*. Of course, his history book is probably one of the first used. That's our favorite graduation gift for mathematics.

A couple of others that I used in this math history course which you might find useful to read and then digest and may be used in your classes: one is #7, which is a three volume set, which is now out in paperback, called *Mathematics: People, Problems and Results*. It's a bunch of short essays on a whole range of topics. From mathematics and art, to Egyptian mathematics, to all kinds of different things, and some biographies in there. And then another related book is called *The Mathematical Experience*, #9, by Davis & Hirsch. I don't know if you could use it at a high school level or not. I think maybe you could give seniors some passages from there. They do a very nice job of clearly writing about some issues like the philosophy of math, like what is mathematics. Is it something that we discover? Some internal/external thing that we look out and pack into and bring back? Or is it something that we create; that we make up a set of rules and follow the rules, like a chess game? There are different schools of thought and different mathematicians have quite different opinions about this. So for students to think about what is this that we do when we do mathematics can be a lot of fun for them. Davis & Hirsch, I think, has one of the clearest descriptions of these kinds of questions and different opinions on them. Any other ideas or resources?

Group dialogue ensues

Any others? If you think of things as we go on, just let us know. We've got a running bibliography here.

There's a video I know that's at least fifteen years old that I thought was fantastic. There were two of them that were absolutely great. *The Square Danced*, one which was dance in a square dance movie on the shapes of squares; all music and shapes and figures. And there was the triangle which was also fantastic. I think that was to classical music. But this one was *The Square Danced*. It's absolutely magnificent. It's like watching a fire; you're spellbound.

There's another one called *Donald Duck in Mathmagic Land*. My seniors ask to see that and I say, "I'm sorry this is an AP course."

Humanistic Mathematics

We're heading into some hands-on activities that we're going to do here. I'm planning a conference a year from this June on what I call humanistic mathematics. When I started thinking about this conference, something I encountered right off was resistance in the math community to this term, humanistic mathematics. I think this is very revealing. Anything that has the word human in it is somehow not mathematics. One of the questions that comes a lot when you use that phrase is "What do you mean by that?" because it's not clear to people what that means. I have three things in mind when I use that term, and I'm going to describe them because they'll help put in perspective the activities that we're doing.

Math as a Human Endeavor

The first thing that I mean by humanistic mathematics is to see mathematics as a human endeavor, something that people do.Part of that involves learning about and understanding the lives of mathematical people as well as seeing mathematics as a cultural product, how it emerges in different cultures. So it's very much what we were talking about in the first two parts of this course. And I think Jacobs' books probably look at a lot of these ideas and, in fact, that is the title, *Math as a Human Endeavor.*

Math and the Other Humanities

But the second part of what humanistic mathematics means to me is to look at the connections between mathematics and other humanities. So what are the connections between mathematics and art?Or mathematics and music? Or literature, for that matter?Or poetry?Because, in fact, the connections are deep and extensive. But recently I was meeting with a woman who was quite involved in quilting—my mother, in the last ten years, has become quite a quilting addict and been in all these shows—and through her I met another woman who has been involved in Islamic quilting and the connections between mathematics and Islamic art. And she gave me this poster which is called a mathematical sampler, and what it is is a series of different quilts, pictures of quilts, but in fact, it's also a puzzle because there are two of each kind of quilt and it's not obvious which ones go together because the coloring and the way they're put together are slightly different. So the puzzle is to see through the colors and see which are essentially the same pattern. And since much of mathematics is really about finding patterns, that's what the underlying theme is to all that we teach. Something like this can be used in the classroom when you're teaching about mathematics or the connections between mathematics and art and what we mean by it.

If any of you are interested in getting a poster like this, on the back of the

bibliography, Margaret Eckels is the name of the person who did it. I think these would be great to have in your rooms.

I think we should work a lot more with this stuff because people get excited about it and can really relate to this. You can teach a lot of mathematics through those forms. I have also been collecting other materials on the connections between mathematics and poetry, for example, and literature, and if any of you are interested, I have some more materials that I could get you.

The Metropolitan Museum of Art has a collection on Islamic art and they lend it to schools on a fifteen day basis if you write them.

That is the person who showed me this, Margaret Eckels. They have a whole booklet of stuff, slides and information about Islamic art. To me, and it sounds like many of you are thinking about interdisciplinary ways of approaching mathematics. To think about the connections between the other humanities is a good way to start. It also becomes very natural to then take a multicultural approach to these areas when you start doing that.

Making Math More Humane

And the third topic in humanistic math—the first is math as a human endeavor, the second is the relationship between math and the humanities—and the third topic is how to teach math more humanely, as one of my colleagues said. And that means really exploring alternative approaches to math education. Many of you are already involved in a variety of alternative approaches, from cooperative learning methods, use of small groups, use of discussion, use of writing in a variety of forms, and something I would like to add to that list that I think is very important is teaching students to learn how to speak mathematics. Many students—this happened to me right through graduate school—can not get up in front of people and talk mathematics. And it's because we don't practice that, and for women, I think, there's a double issue of finding their voices. So I'm really working hard in the classroom at helping students develop their mathematical voices and their voices in general. And I think there are many techniques you can use for doing that. So all of those things, I think, bring together this idea of humanistic mathematics which is connected to the history of mathematics which is connected to women in mathematics.

Mathematics & Geometry in Nature

Now what I want to do is describe a few kinds of activities that I think illustrate a lot of these topics and points. To begin with, something I did in an alumnae college course, I wanted people to begin to think about the relationship between mathematics and nature and its relationship to geometry. In particular, how does math grow out of, and express itself in, nature.

And so the first thing that you can have students do in talking about geometry or math and nature is to have them go out and start collecting things that exhibit different kinds of shapes, what kinds of shapes can they find in nature. Now what kinds of shapes do you think they'll come up with if you just send them out to collect things? What kinds of things would they find?

Circle.

Okay, they'd find lots of circles, in what?

An orange.

What other kinds of shapes would they come up with?

Polygons.

What kinds of polygons?

Triangles.

Where would they find a triangle?

In a leaf. In a Ginko tree.

Yes, some trees are shaped in a triangular form.

They could find polygons in a spider web.

Right.

The honeycomb of a beehive. Hexagon.

Spirals.

Spirals in a shell, yes. Parallel lines in crystals.

Pentagons. Any idea where they might find pentagons?

Stop signs.

Right. What about in nature?

Slice an apple or pear crosswise. Sand dollars. Starfish.

Starfruit, for that matter. It's amazing how pentagons are really quite prevalent there are a lot of places where they occur. The other thing that's really interesting to have students think about is what shapes occur in nature which are man-made or human made, for example, it's very hard to find a square in nature. Most of the ones you've given me are natural shapes, right, but can you think of where a square occurs in nature?

Crystal might come in cubes.

That's interesting. They're hard to find. There is a wonderful passage that I want to read to you to make them think about shapes and their relationship to the culture. In our culture the squares and lines are very much dominant in our culture. We sort of revere the line as the shortest distance between two points and almost everything we build has a rectangular shape. We really think in terms of right angles. Our buildings, everything we live in and all that, is mostly about right angles. But clearly other people don't live in rectangular spaces; a lot of people live in circular things. And it's interesting to think about how that affects the way they perceive the world.

There are two passages in this book, *Ethnomathematics*, and the first one is about the line and how much that's built into our Western culture right now. And that's the passage from the book, *The Mathematical Experience*, but there's another passage opposite it from a book called *Black Elk Speaks*.

And I want to read this passage to you, and it's about the circle as opposed to the line. Just sit back for a minute and relax and listen to this passage:

> I am now between Wounded Knee Creek and Grass Creek. Others came too and we made these little gray houses of logs that you see. And they are square. It is a bad way to live for there can be no power in the square. You have noticed that everything an Indian does is in a circle and that is because the power of the world always works in circles, and everything tries to be round. In the old days, when we were a strong and happy people, all our power came to us from the secret hoop of the nation. And so long as the hoop was unbroken, the people flourished. The flowering tree was the living center of the hoop and the circle of the four quarters nourished it. The East gave peace and light. The South gave warmth. The West gave rain. And the North, with its cold and mighty wind, gave strength and endurance. This knowledge came to us from the outer world with our religion. Everything the power of the world does is done in a circle. The sky is round and I have heard the earth is round like a ball and so are the stars. The wind in its greatest power whirls. Birds make their nests in circles for theirs is the same religion as ours. The sun comes forth and goes down in a circle. The moon does the same, and both are round. Even the seasons form a great circle in their changing and always come back again to where they were. The life of a man is a circle from childhood to childhood and so it is in everything where power moves. Our teepees were round like the nests of birds and these were always set in a circle, the nation's hoop, a nest of many nests where the Great Spirit meant for us to hatch our children. But the white men have put us in these square boxes. Our power is gone and we are dying. The power is not in us anymore. You can look at our boys and see how it is with us. When we were living by the power of the circle in the way we should, boys were men at 12 or 13 years of age, but now it takes them very much longer to mature. Well, it is as it is. We are prisoners of war while we are waiting here, but there is another world.

And that's such a different perspective about the meaning of the circle and how that influence comes from a cultural belief system and influences them.

I live on a vegetable farm and I thought about everything that grows on that farm and everything that I could think of is round; that in nature that's such a predominant shape. I mean, what vegetables do you know? There's peas, squash, cucumbers, tomatoes, melons, beets—cross-sectioned—not spheres. Even eggplant has a cross-section, cauliflower...

Celery grows up in a round form.

Yes, that's true. And it's interesting just what a dominant shape that is and yet, because so much of our life is about rectangles and squares, we just don't

even notice them. We don't see that. So it's a good way to begin thinking about mathematics or geometry, is to sort of just pay attention to how it exists in our lives and what shape it takes.

Symmetries Everywhere

I'd like to investigate symmetries, or strip patterns as they're called. Obviously, by symmetries we mean repeating patterns, and you're looking at some examples on the first page there. Now symmetry is a truly wonderful topic to explore and talk about for a number of different reasons. It's a topic that crosses many different fields. It comes up in science; it comes up in music; it comes up in math; it comes up in quilts; it comes up all over the place. And one of the things that I think is fun to do with students is have them think about where they see symmetries. So where might they come across symmetries?

Nature. In their own bodies. Butterflies. Architecture. Beetles. The windows; the latches are both on the inside.

What else? Most of them are probably wearing symmetries of some sort, some kinds of patterns or in headbands, or on their briefcases or bags or something like that. So if they look, they'll see quite a lot of symmetries all around them.

Their shoes.

Yes, a pair of shoes, or in pottery, something like that. So the first thing to do is notice where these different symmetries occur. What we want to do now it to categorize all the kinds of symmetries there are. The first point to be made about categorizing symmetries is that there's more than one way to do it, but what we're going to talk about is the Western way of categorizing symmetry. It's one way of doing it that's useful because it lends itself to mathematics; a sort of systematic way of saying how many different symmetries there are and things like that, but there are many other ways to do it. You could talk about what purpose symmetry serves. Or where it occurs.

But we won't be concerned about whether, in fact, what you're looking at came from Brazil, Persia, England, Greece...they came from all over the place and you're not, at this point, concerned about where they come from, but in what ways are they similar or different from each other. So we're looking at a kind of internal logic in classifying them on those terms.

Now, how many different kinds of symmetries are there? Well, in order to ask that question, you need to ask, What do we mean by symmetry? We mean if you take some kind of strip with a design on it that repeats itself, what kind of motions can you do to that strip so that it looks exactly the same after you've done the motion? Those motions are called isometries. They're motions that preserve the pattern essentially so that if you do that motion and you put your strip back down, it will look exactly the same as when you

started. What would be an example?

Turn it over.

So if you've got something...let's take something like the letter **p** or something and you just repeat it, that's an example of a strip symmetry. It's a repeating pattern and what you just pointed out is that the motion that we could do to this is just move it over and put it back down and it looks identical to before. So one of the motions is called a translation. And that's just moving it over. And this is an example of a symmetry which has translation as its isometry. What's another kind of symmetry.

Reflection

Reflection; flipping it over, is that what you're describing? What would it look like? What would a strip pattern look like that had a reflection symmetry? It depends on which kind, right? There are two kinds of reflections you could be doing. So this is a vertical reflection; if you take a vertical line and you reflect through that line, you get the exact same strip back again. So you could have a reflection, and as we just saw there, we could have a vertical or a horizontal one. And let's picture what a horizontal symmetry would look like. If this was our union, you would flip it over and you'd get that. Is that right? Does this, by the way, have any other symmetries?

Translations, Rotations, Reflections

It also has a translation, right. So some patterns have more than one isometry associated with it. What's another kind of symmetry you can have?

A rotation; about how many degrees?

For a strip?

Yes, for a strip. What was that?

180.

It can only be 180, in fact. Why is that? Why can't you have a 90 degree rotation on a strip? Yes, because if you start a strip like that and rotate it, you're not on your strip anymore. Now what would it look like if we started with our **p** as the basic unit and we rotate it 180 degrees? What would you get?

d.

Yes. Can you see that? So that's a symmetry that's a rotation. Does that have any other symmetries? It also has a translation except the unit in this case is this whole thing, a **p/d** if you do a translation. So you have rotation. Other there any other isometry motions? This is the least obvious. Yes, where you do a reflection and a translation together, that's called the glide so that's equal to the reflection plus translation. Now that's one of the harder ones. So say you start with a **p** and you were to translate that over and it was

reflected in a horizontal, do you know what that would look like?

It would be a **b**.

That's right. I always get this wrong. I keep wanting to do a rotation, but it's not. You move it and you reflect it through the horizontal axis, you get a **b** when you do that. You flip it over and you get a **b**. So that's a glide reflect.

Now, here's a really interesting question. Those are the basic isometry motions. Now, one question you can ask is how many different possible symmetry patterns can we get just using those four motions, but you can use them in combination. So you could do a reflection plus a rotation plus a translation or something like that. So you can do any combination of them. And you want to do all the possible strip symmetry designs you could do, just using those four groups in any combination. What I'd like you to do it is just guess, totally off the top of your head, but just around how many different possibilities you could come up with to make the designs. Any idea?

24, 4^4, 2^4.

I just want to know how many truly different kinds of designs you can get so if by doing translation and reflection you get the same as two rotations, then I won't count those as two different things. I want to know how many unique designs you could get. Do you see what I'm saying.

So some orders of some choices are going to produce the same result?

Yes, for one thing, if you do three translations, what's that equivalent to? One translation, right? Or if you do a translation and a horizontal reflect together, what do you get? You get a glide, right? So you might not have as many, right? Well, get this, of all the possible, possible strip designs in the world that were ever created, there are only seven possibilities. Isn't that amazing? There are only seven distinct possibilities.

Creating Symmetries

One thing you could do—this is really a great project for students—is to not tell them this, but have them do experiments and try and have them create the different symmetries. You could just start with the letter **b** to see how many different things they can get. If we had more time, I'd have you do that. They can have a hypothesis, they can test it, they can experiment with it, and try and come up with those different categories. Then once they do that, even knowing the categories which are listed on the back page, there's different ways of describing it there.

I'm passing out a batch of strip designs. Take one and pass it on. When I get to the end I want to put clips on them again. You're going to have a lot of these. Sixteen total. Some of them are the same, in fact, you have at least two of each category. You've got to figure out which ones are essentially the same design, and figure out what the symmetry group is. Now, I don't have time to get into it right now, but there is a lot of math behind all this. If any of you teach about groups...do you still teach groups in high school?

Not much.

Not much. Well, if you do teach with groups, it turns out that each one of these seven symmetry categories forms a group. It has an identity element; you have inverses; they are associative. It's a great way to teach that stuff.

In Conclusion

I wrote down three different books on your bibliography that talk about the symmetries which I have here, from which I copied these strip designs, but there are lots of sources. Just go to an art library or something like that. I've just begun to scratch the surface of the kinds of things you can do in a classroom with symmetries.

This is not easy, that's right. I have the answers.

Do you have an answer sheet so we can check it out?

No, no. You can come to me later or you can write to me. For the benefit of participants, answers are attached. For others, a source for the activity is listed on the answer sheet. They have the same symmetries. This one only has translation, right. If you reflect it horizontally, you don't get the same pattern back, right. Or, if you reflect it this way, you won't get the same pattern back. It only has one symmetry, one isometry you can do to it, which is translation. So find the other strip pattern which has only one symmetry.

What is really nice at the very end of studying things like this is that you can bring it back to a cultural context. In fact, one of the things that Marcia Ascher talks about in *Ethnomathematics* is a culture that did symmetry work in the rafters of a community building. And it was a skill that was passed on to one or two people in each generation. And there's a whole religious significance to the kinds of symmetries they did and whole rituals behind it. Other people work in symmetries for different purposes and it had a whole different meaning. So, again, just to place it in those different contexts, there's this overlap from cultures as old as Ancient Egypt to modern America. ❖

Bibliography

Aaboe, Asger, *Episodes from the Early History of Mathematics*, L.W. Singer, New York, 1964.

Abir-Am, Pnina, and Dorinda Outram, *Uneasy Careers and Intimate Lives: Women in Science, 1978-1979*, Rutgers University Press, New Brunswick, N.J., 1987.

Albers, Donald, and G.L. Alexanderson, *Mathematical People*, Birkhauser, Boston, 1985.

Berggren, J.L., *Episodes in the Mathematics of Medieval Islam*, Springer-Verlag, New York, 1986.

Boyer, Carl, *A History of Mathematics*, Princeton University Press, Princeton, NJ, 1968.

Campbell, Douglas, and John Higgins, eds., *Mathematics: People, Problems, Results*, Wadsworth International, Inc., Belmont, California, 1984.Campbell,

Paul, and Louise Grinstein, eds., *Women of Mathematics: A Bio-Bibliographic Sourcebook*, Greenwood Press, New York, 1987.

Dauben, Joseph, *The History of Mathematics from Antiquity to the Present. A Selective Bibliography*, Garland Press, New York, 1985.

Davis, Philip, and Reuben Hersh, *The Mathematical Experience*, Birkhauser Boston, Cambridge, MA, 1981. (Paperback: Boston: Houghton Mifflin, 1982.)

Greenberg, Marvin J., *Euclidean and Non-Euclidean Geometry*, W.H. Freeman and Co., San Francisco, 1980.

Hall, Tord, *Carl F. Gauss*, translated by Albert Froderverg, The M.I.T. Press, Cambridge, MA, 1970.

Hardy, G.H., *A Mathematician's Apology*, The University Press, Cambridge, England, 1940.

Hardy, G.H., *Twelve Lectures on Subjects Suggested by His Life and Work*, The University Press, Cambridge, England 1940.

Kline, Morris, *Mathematical Thought from Ancient to Modern Times*, Oxford University Press, Oxford, England 1972.

Kline, Morris, *Mathematics, The Loss of Certainty*, Oxford University Press, New York, 1980.

Mueller, Ian, *Coping with Mathematics (The Greek Way)*, Morris Fishbein Center for the Study of the History of Science and Medicine, No. 2, Chicago, IL, 1980.

Osen, Lynn, *Women in Mathematics*, The MIT Press, Cambridge, MA, 1974.

Perl, Teri, *Math Equals*, Addison-Wesley Publishing Company, Reading, MA, 1978.

Ranganathan, Shiyali, *Ramanujan, The Man and the Mathematician*, Asia Publishing House, New York, 1967.

Wilder, Raymond, *Mathematics as a Cultural System*, Pergamon Press, New York, 1981.

Wilder, Raymond, *The Evolution of Mathematical Concepts*, J. Wiley and Sons, New York, 1968.

Zaslavsky, Claudia, *Africa Counts: Number and Pattern in African Culture*, Prindle, Wever & Schmidt, Boston, 1973; paperback, Lawrence Hill, Westport, CT, 1979.

Articles

Berndt, Bruce C., "Srinasvasa Ramanujan," The American Scholar, Vol. 58, No. 2, Spring 1989, pp. 234-244.

Grabiner, Judith, "The Centrality of Mathematics in the History of Western Thought," Mathematics Magazine, Vol. 61, No. 4, Oct. 1988.

Holmes, Oliver Wendell, "Descartes" from *Calculus and Analytic Geometry*, by George Simmons. McGraw Hill, 1985.

Langer, Rudolph, "The Life of Leonard Euler," Scripta Mathematica, Vol. 3, No. 1 and 2, 1935, Jan. and April.

Steen, Lynn A., "The Science of Patterns," Science, Vol. 240, April 29, 1988.

Weil, Andre, "History of Mathematics: Why and How," *Collected Papers*, Vol. III, Springer, New York, 1980.

Symmetry Books

Symmetries of Culture by Dorothy Washburn and Donald Crowe, U. of Washington Press, Seattle, 1988.

Patterns of Symmetry by Marjorie Senechal and George Fleck, U. of Mass. Press, Amherst, 1977.

Handbook of Regular Patterns by Peter Stevens, MIT Press, Cambridge, MA, 1980.

Other Books

Ethnomathematics: A Multicultural View of Mathematical Ideas by Marcia Ascher. Brooks/Cole Pub. Co., Pacific Grove, CA, 1991.

The Golden Section by Garth Runion. Scott, Foresman and Co., IL, 1972.

MATH SOURCES

Fantasy & Symmetry, The Periodic Drawings of M.C. Escher, Caroline H. MacGillavry, Harry N. Abrams, Inc., New York, 1976.

Introduction to Geometry, H.S.M. Coxeter, John Wiley and Sons, New York, 1969.

Transformational Geometry, Richard G. Brown, Dale Seymour Publications (paper), Palo Alto, CA, 1973.

Geometric Design, Dale Seymour Publications (paper), Palo Alto, CA, 1988.

Introduction to Tessellations, Dale Seymour and Jill Britton, Dale Seymour Publications (paper), Palo Alto, CA, 1989.

Tessellation Teaching Masters, Dale Seymour, Dale Seymour Publications, Palo Alto, CA, 1989.

Tilings and Patterns, Branko Grunbaum and G.C. Shepahard, W.H. Freeman and Co., New York, 1987.

Module, Proportion, Symmetry, Rhythm, Gyorgi Kepes, G. Braziller, New York, 1966.

Geometry, An Investigative Approach, P.G., Clemens, S.R. O'Daffer, Addison Wesley Publishing Co., Menlo Park, CA, 1977.

Symmetry Discovered, Joseph Rosen, Cambridge University Press, Cambridge, England, 1975.

The Geometry of Art and Life, Matila Ghyka, Dover Press, 1946.

Patterns of Symmetry, G. Senechal and M. Fleck, Boston University Press, MA 1977.

The Sense of Order, E.H. Gombrich, Cornell University Press, Ithaca, NY,1979.

Pattern: An Historical Panorama, William Justema, New York Graphic Society Books, 1976.

Creating Escher-Type Drawings, E.R. Ranucci and J.L. Teeters, Creative Publications, 1977.

The Graphic Work of M.C. Escher, M.C. Escher, Hawthorn Books, Inc., 1960.

"The Plane Symmetry Groups: Their Recognition and Notation," Doris Schnattschneider, *The American Mathematical Monthly*, 85 (June-July 1978): 439-450.

Symmetry, Hermann Weyl, Princeton University Press, Princeton, NJ 1952.

Pattern: Its Structure and Geometry, Richard Padwick and Trevor Walker, Cleofrith Press, Sunderland, England, 1977.

The Mathematics of Islamic Art, (teachers packet), Jane Norman and Steph Stahl, The Metropolitan Museum of Art, New York, 1979.

**Patterns East & West*, Jane Norman, Margit Echols, Harry Bixler, Ph.D., Steph Stahl, The Metropolitan Museum of Art, New York.

Art Sources

The Miraculous Journey of Mahomet, commentary by Marie-Rose Seguy, George Braziller, New York, 1977.

A King's Book of Kings, Stuart Carey Welch, The Metropolitan Museum of Art, New York, 1976.

Early Muslim Architecture, K.A.C. Creswell, Penguin Books Ltd. (1958), reprinted in Lebanon (1968), Lebanon Book Shop, Bliss St., Beirut.

Art of Islam, Carel J. Du Ry, Abrams, New York, 1970.

The Art and Architecture of Islam 650-1250, Richard Ettinghausen and Oleg Grabar, Viking Penguin, Inc. (paper), New York, 1987.

Islamic Art, David Talbot Rice, Oxford University Press, Oxford, England 1975.

The Formation of Islamic Art, Oleg Grabar, Yale University Press (paper), New Haven, 1978.

Islamic Ornamental Design, Claude Humbert, Hastings House Publishers, New York, 1980.

Islamic Painting, Richard Ettinghausen, Marie Swietochowski, *The Metropolitan Museum of Art Bulletin*, (Fall 1978), New York.

Islamic Art, members of the Department of Islamic Art, *The Metropolitan Museum of Art Bulletin*, New York.

Islamic Glass, Marilyn Jenkins, *The Metropolitan Museum of Art Bulletin*, (Fall 1986), New York.

Islamic Pottery, Marilyn Jenkins, *The Metropolitan Museum of Art Bulletin*, (Spring 1983), New York.

The Decorative Art of Arabia, Prisse D'Avennes, Paris 1869, reprinted by Portland House, New York, 1989.

Arab Geometrical Pattern & Design, J. Bourgoin, Paris 1879, reprinted by Dover Publications (paper), New York, 1973.

Handbook of Regular Patterns: An Introduction to Symmetry in Two Dimensions, Peter Stevens, MIT Press,Cambridge, 1980.

Japanese Decorative Style, Sherman Lee, Cleveland Museum of Art, Cleveland, 1961.

Puzzle Poster. *Patchwork Sampler*, Margit Echols, Rowhouse Press, P.O. Box 20531, New York, New York 10025, (212) 662-9604.

Barbara Lazarus

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BARBARA LAZARUS HAS BEEN ASSOCIATE Provost for Academic Projects and Adjunct Associate Professor of Educational Anthropology at Carnegie Mellon University since 1985. In that position, she has been responsible for creating and coordinating programs to support women and minorities in science and engineering. Today she is returning to Wellesley where she worked as Associate Dean and Director of the Center for Women's Careers. In addition, she has administered programs in research and academic advising at the Educational Development Center in Newton, Massachusetts, and at the University of Massachusetts College of Education. She has also taught at the University of Connecticut.

Ms. Lazarus is an international consultant on Women, Education and Work. In this capacity she has traveled widely in England and throughout Asia. Her major research interests are women in non-traditional occupations, and the career impacts of illness. Today she will be working with us on *The Math, Science, and Engineering Major in College: What Turns Them Off and How Can We Retain Them?* ❖

The Math, Science, and Engineering Major in College

~What turns them off and how can we retain them?~

Barbara Lazarus

IN THIS WORKSHOP, participants will have a chance to engage in a discussion of the particular problems faced by undergraduates and graduate students at the university, including entry into the fields of science and engineering, retention through the undergraduate years, deciding to enter graduate school, completing an advanced degree and seeking careers in the academy. Participants will have the opportunity to review the problems confronting college and university administrators as well as strategies they are developing to overcome them. An effort to develop new programs and strategies bridges the high school-college continuum. The workshop will be conducted as a seminar based on reading several short texts.

REQUIRED READING: *AAAS Presidential Lectures: Voices from the Pipeline*, Sheila E. Widnall, Science, Vol. 241.

Out of the Classroom: A Chilly Campus Climate for Women, Roberta Hall and Bernice Sampler, Association of American Colleges, Washington, D.C.

"The Math, Science and Engineering Major in College: What Turns Them Off and How Can We Retain Them?" Barbara B. Lazarus, Associate Provost for Academic Projects, Carnegie Mellon University.

ATTACHED ARE THE CASES STUDIED.

Unacceptable Behavior

These fictional cases are based on real experiences of women college students at a number of universities. The questions raised are used as training exercises for teaching assistants, members of the faculty, and students, to discuss together how to improve the environment for women in the sciences and mathematics.

The topic of today's discussion will focus on sharing some of their conclusions and yours. Our major questions for today will be: How can the high schools work together with the colleges to improve the environment? What can colleges and universities learn from your experience? How can you prepare students to meet these challenges? Should you?

These fictional stories are based on real experiences of Carnegie Mellon students. This is sexual harassment. If this happens to you, talk with a member of the Human Relations Commission.

Alice Newton: Losing in Sophomore Physics

The only woman in the sophomore physics course, Alice Newton finds herself the unwilling social interest of her TA, Tom Kepler. Tom continually comments on how attractive he finds Alice and calls her in her dorm to ask if she needs help with her work. Alice is very uncomfortable with Tom's attention, and after much soul searching, goes to see Professor Brahe who teaches her course. Alice respects Professor Brahe a great deal and feels that he will be able to address her concern.

Professor Brahe finds Alice's situation disquieting, but suggests that since less than half of the semester is left, she should "tough it out" for the remainder of the course. "You have to be tough to succeed in physics" he explains. "And while you could take this case on, it would probably cause you more trouble than it's worth." He further explains: "Tom is lonely, being so far away from his family and friends. The male/female ratio makes it very difficult for Tom to have a social life. If he's not allowed to pay attention to students, who will he meet?"

Alice reports that she continues in the course, but feels physically sick every time she goes to class. She doesn't do as well as she usually does and switches majors at the end of the semester.

Issues to Consider: Is toughness necessary for success in a non-traditional field such as physics? Does talking to someone necessarily create trouble for the victim? What else could Professor Brahe have said? What else could Alice have done?

Betty Von Neumann: A Mathematics Ph.D. Student

Betty Von Neumann is a first year Ph.D. student in mathematics. After several months in the program, she learns that only two of ten women Ph.D. students have passed their qualifying exams for the last several years. Betty finds this disquieting and wonders if it relates to implicit or explicit discrimination. She speaks to several faculty members about her concerns and is told by one that she is "paranoid". Her advisor reassures her that "the women were not the only ones who failed and that each of them deserved to fail." She doesn't know who to turn to and finds that she is distracted by these concerns in doing her own work.

ISSUES TO CONSIDER: Should the perception of prejudice be discounted? What are alternative responses to her concern? Are there alternative things Betty could have done to have addressed her fears?

Paula Von Karman: A Senior Engineering Student

Paula Von Karman is a senior in Mechanical Engineering with a very strong academic record. She has undertaken undergraduate research, is confident of her abilities, and is interested in an opportunity to undertake a Ph.D. program. She has a strong commitment to her profession and an equal commitment to her personal life. In late fall of her senior year she has become engaged to another student; they plan to marry during early summer.

Because Paula is a good student, she has been encouraged by many people to think about going to a top graduate school. But when she announced her upcoming marriage, several faculty members suggested that, given her commitment to marriage, she should consider attending a less competitive school "in order to better balance her life". Professor Thoreau, a faculty member with whom she is particularly close, told her of his own disenchantment with the "tough" environment of a "competitive" school and the stress and strain that a rigorous program had on his life.

While Paula feels "somewhat relieved", she is also confused about the impact of such a decision on her career in the long run. Professor Thoreau reassures her that as long as she does not want to teach at a "top" school, attending a less rigorous school will not have a negative impact on her long run opportunities. Paula chooses to apply to less rigorous schools.

ISSUES TO CONSIDER: Is there a relationship between a rigorous and competitive curriculum and nurturing? How is stress in a new marriage directly related to a rigorous and competitive graduate school environment? Should someone advise somebody to something which one has not done oneself? Are there other ways Professor Thoreau could have responded to Paula? Are there other ways Paula could have responded to Professor Thoreau? ◆

Jan Serie

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JAN SERIE IS CURRENTLY AN ASSOCIATE PROFESSOR of biology at Macalester College. She has received grants to develop techniques in research and to equip laboratories, and is director of a campus program for curriculum development and student-faculty research. Her research interests focus on graft survival, diabetes, and immune recognition across species barriers. She is resident scientist on KTCA-TV's NEWTON's APPLE, and a member of the Ford Grant Steering Committee. Her public education projects include anatomy for attorneys and continuing legal education.

Today she will be working with us on techniques of science teaching that accommodate diverse learning styles.❖

Techniques for Teaching Science

~ *That accommodate diverse learning styles* ~

Jan Serie

IN *WOMEN'S WAYS OF KNOWING*, Belenky, et al. propose several characteristics of an educational environment that supports the intellectual development of women. Recently several models for teaching have emerged which, although not derived from Belenky, nonetheless address many of the educational goals outlined. The computer program BioQuest, based on the three P's, problem posing, problem solving, and persuasion, can be well used in lecture sessions, computer laboratories, and "wet labs".

WHAT FOLLOWS IS AN OUTLINE OF THE WORKSHOP.

SCIENCE EDUCATION FOR GIRLS
COLLABORATIVE LABORATORY INVESTIGATION

Overview of Workshop

1. Discussion of workshop goals.
2. Presentation of some ideas from *Women's Ways of Knowing: The Development of Self, Voice and Mind* by Belenky, Clinchy, Goldberger, and Tarule.
3. Active involvement in collaborative laboratory investigation using computer simulations of real experiments (BioQuest Program).
4. Discussion

Workshop Goals

1. To learn more about some of the literature on women's learning styles and intellectual development.
2. To entertain some new ideas about teaching styles and course design, which may enhance science participation and learning by female students.

3. To try out a model for collaborative learning in the laboratory.
4. To analyze this approach in terms of your own teaching experience, and share your opinions about its usefulness in educating girls in science.

Women's Ways of Knowing

Purpose of the Study: The project began in the late 1970s. As psychologists interested in human development, we had spent a large part of our professional lives studying the intellectual, ethical, and psychological development of adolescents and adults in educational and clinical settings. We became concerned about why women students speak so frequently of problems and gaps in their learning and so often doubt their intellectual competence. We had also become aware of the fact that, for many women, the "real" and valued lessons learned did not necessarily grow out of their academic work, but grew out of relationships with friends and teachers, life crises, and community involvements. Indeed, we observed that women often feel alienated in academic settings and experience "formal" education as either peripheral or irrelevant to their central interests and development.

The authors focused on studying the intellectual capacities, modes of thought, and aspects of intelligence which are preferred by, and become highly developed in, women.

Method of Study: All information was gathered in open-ended interviews. The authors did not wish to impose preconceived notions about what themes might be important to women. They developed a series of open-ended interview questions about background, self-description, the role of gender in the women's lives, relationships, moral dilemmas, education, ways of knowing, and hopes for the future.

They conducted extensive interviews with 90 students from six academic institutions chosen for diversity. The students within the colleges were also chosen for diversity in race/ethnicity, year in school, socioeconomic background, academic performance, and degree of engagement with institution.

The authors also interviewed 45 women who were seeking education about child-rearing from a social service agency. The authors call this the "invisible college".

Results: The authors refrained from distilling the results into tables and charts, and instead presented their findings in narrative form. It is, therefore, impossible to do justice to the study in summary form. However, in overview, the authors identified five "stages" of intellectual development common to women. (I put "stages" in quotes because the authors do not claim that these stages lead linearly from one to another.)

The Stages Are

- Silent Knowledge
- Received Knowledge: Listening to the Voice of Others
- Subjective Knowledge: Listening to the Inner Voice
- Procedural Knowledge: Knowledge by Process
- Separate Knowing
- Connected Knowing
- Constructed Knowledge: Integrating the Voices

Separate and Connected Knowing in the Science Classroom

Separate Knowing:

> "Presented with a proposition, separate knowers immediately look for something wrong — a loophole, a factual error, a logical contradiction, the omission of contrary evidence."
> "You take a point of view and then you address the points of view that might most successfully challenge your point of view. You try to disqualify those."

Separate Knowing is Characterized by:

- Playing the "doubting game"
- Learning through opposition, trying to find a flaw
- Debate
- Analysis based on rules
- Authority resting on reasoning ability rather than position or status
- Trying to "weed out" the self from an argument or position
- Excluding feelings, intuition, emotion from analysis and expression

Most of the women who leaned heavily toward separate knowing were attending or had attended traditional, elite, liberal arts colleges. However, most women who had mastered this way of knowing or who knew they were expected to master it saw it as a game, disconnected from their inner selves.

"The problem is that I don't feel terribly strongly about one point of view, but that point of view seems to make more sense. It's easier to write the paper, supporting that point of view than the other one, because there's more to support it. And it's not one of my deep-founded beliefs, but it writes the paper."

"It's just rhetoric. It's just a game. It doesn't prove anything."

In science, separate knowing is dominant in the review of journal articles, the question period at the end of a scientific presentation at a meeting, the analysis of data to make sure no interpretation has been left out, and the design of experiments to insure that the experiment will unambiguously answer the question being asked.

Connected Knowing:

> "If you listen to people, you can understand why they feel the way they do. There are reasons. They're not just being irrational."
> "Negative judgments seem (to a connected knower) like a violation of connection by the assertion of superiority."
> "They can make moral judgments, but did not wish to impose these judgments on others."
> "When someone said something they disagreed with or disapproved of, their instinct was not to argue but to look at it from that person's point of view, see how they could say that, why they think that they're right, why it makes sense."

Connected Knowing is characterized by:

- Playing the "believing game"
- Learning through caring, empathy
- Interviews
- Analysis based on intimacy, connection to speaker
- Authority resting on personal experience
- Trying to find and better define the self from an argument or position
- Including feelings, intuition, and emotion in analysis and expression

In science, connected knowing is used (or should be used) in defining and narrowing the question being asked, in communications about the set-up and performance of the experiment, in group analysis of data, in proof-reading papers, in lab meetings when newly formed ideas emerge.

Placing More Emphasis on Connected Knowing

The educational system (at least at the college level) supports the development of separate knowing well, but fails to provide enough support for connected knowing.

How can connected knowing be supported?

From *Women's Ways of Knowing*: Believing in Students' Intellectual Ability from the Start:

> Because so many women are already consumed with self-doubt, doubts imposed from outside seem at best redundant and at worst destructive. The doubting model, then, may be peculiarly inappropriate for women.
>
> Women reported the experience of being doubted as debilitating rather than energizing.

Confrontations, doubting, and criticism are difficult for women to handle, especially early in their academic careers, because of internalized

messages about failure and women's orientation toward connection. Silence about ability negatively impacts girls because it does nothing to counter internalized negative messages.

Student Possesses the Skills to Succeed

TWO EXAMPLES:

> The professor marched into the lecture hall, placed upon his desk a large jar filled with dried beans, and invited the students to guess how many beans the jar contained. After listening to an enthusiastic chorus of wildly inaccurate estimates, the professor smiled a thin, dry smile, revealed the correct answer, and announced, 'You have just learned an important lesson about science. Never trust the evidence of your own senses.' I remember feeling so small and scared, and I did the only thing I could do. I dropped the course that afternoon, and I haven't gone near science since.

In this example, the professor stripped the students of the tools of measurement (he would not let them count the beans) and yet expected a reasonable answer. The student felt powerless without the only tool she had available, her perceptions.

> The teacher came into class carrying a large cube. She placed it on the desk in front of her and asked the class what it was. They said it was a cube. She asked what a cube was, and they said a cube contained six equal square sides. She asked how they knew that this object contained six equal square sides. By looking at it, they said. 'But how do you know?' the teacher asked again. She pointed to the side facing her and, therefore, invisible to the students; then she lifted the cube and pointed to the side that had been face down on the desk and, therefore, also invisible. 'We can't look at all six sides of a cube at once, can we? So we can't exactly *see* a cube. And yet, you're right. You know it's a cube. But you know it not just because you have eyes but because you have intelligence. You invent the side you cannot see. You use your intelligence to create the 'truth' about cubes.

In this example, the teacher is reinforcing the students' image of themselves as intelligent beings, already possessing the tools required to succeed. The cube "model" invented by the students' minds is no different from the models of science we invent based on some data and a lot of intelligence.

The "weed out" course is the antithesis of this approach, where students are made to feel inadequate because the material is overly difficult. This is harder on women than men, because of negative cultural images of women's intelligence which most young women have internalized.

Information Related to Personal Experience

> Most of the women were not opposed to abstraction as such. They found concepts useful in making sense of their experiences, but they balked when the abstraction preceded the experiences or pushed them out entirely. Even women who were extraordinarily adept at abstract reasoning preferred to start from personal experience.
>
> I don't like getting things totally out of a book. I really like having some real experience of it myself. In the most exciting kind of learning, people are allowed to go right down to rock bottom and really look at these concepts and find out what their experience has been with it, what they know about it.

Science taught in a linear, distilled fashion, omitting the relationship to the lives of people, does not appear to fit the learning styles of most women. Scientists do not learn this way, and yet we expect students to sit passively and follow along the linear path we have chosen for them.

Evaluations Constructed with Student Input

Evaluation, like teaching style, is peculiar to the individual teacher and therefore skewed toward her/his values. The method of evaluation in a course is so much more than simply a way of assigning grades. It is a clear message about what is important. A diversity of points of view exist in a class about what is important learning. Some of this should be tapped and be incorporated into the structure of evaluation instruments. Even instrument styles (e.g. oral exams, multiple choice exams, papers) favor one learning style over another and, if all evaluations in a course are the same type of instrument, students whose learning styles fit this instrument will be at an advantage.

The intrument must be willing to take intellectual risks in front of the students. The following passage is about a professor who presented an analysis of *The Turn of the Screw* in class and then invited the class "to rip into" his analysis (a separate knowing approach). The woman relating the tale did not respond.

> It is easy to feel compassion for this beleaguered man. He had probably toiled much of the previous night over his interpretation. He is excited about it and imagines that the students too will get excited. Instead, he sees rows of bowed heads, hears only the scratching of twenty-five pencils. The teacher does not wish to deposit words in the students' notebooks, but the students insist upon storing them there. They treat his words as sacrosanct. He cannot understand why they will not risk a response. But the teacher himself takes few risks. He composes his

thoughts in private. The students are permitted to see the product of his thinking, but the process of gestation is hidden from view. The teacher asks the students to take risks he is unwilling to take.

In order for women to learn how to develop ideas from embryonic stages to maturity, they need to see this modeled for them. Teachers rarely "think out loud" and yet this provides an excellent lesson on how to think (blind alleys and all) *and*, perhaps more importantly in the case of women students, it demystifies the process of thinking and demonstrates that it's not so hard to do. The willingness to be vulnerable builds trust and connects teacher to student.

Midwife teachers do not administer anesthetics.

Connected teaching is a process by which student development is encouraged by the teacher. This is the antithesis of a model where teachers pour knowledge into the heads of students. Rather, in the midwife model, teachers help students construct knowledge according to their own emerging world views. Teaching midwifery is a dialogue—listening to the student is as important as talking.

Supporting Intellectual Growth

A large part of teaching is creating the environment in which learning occurs. A culture which supports intellectual growth has several characteristics: trust between members, no power-plays by instructor, sincere tolerance of a diversity of perspectives, good-faith efforts to understand one another, nurturance of newly formed ideas.

Supporting Separate and Connected Knowing

GROUP WORK: Both separate and connected knowers learn through interactions with others. Separate knowers can be expected to operate based on perceived group rules and to expect ideas to be challenged. Connected knowers will base their interactions on their relationships with group members and may not feel comfortable intellectually in a group until they know the group members fairly well. Therefore, the establishment of groups which stay together for a while may help connected knowers (provided the group members work well together) and will probably not hurt separate knowers.

Groups of three seem ideal. That way, students are not stuck with a "partner" with whom they have difficulty, yet the groups are small enough that everyone must participate.

Group process should be made explicit. Groups should not simply be put together and expected to function. Roles may be assigned (e.g. recorder, devil's advocate, consensus builder, active listener, conflict resolver) or natural roles may be reported after the fact. Care must be taken to help when

very dominant or very passive people are involved. It may help to have the class talk about how they want to be treated in groups before they begin working together. The class may also want to design a mechanism for dealing with groups that don't work well together.

Investigative Format for Labs: Labs that have a completely investigative format require students to work together to ask a question, design an experiment, perform it, analyze the data, and present it in written or oral form. Any one aspect of this process can be used in investigative exercises. In my experience, the best investigative formats supporting connected knowing have the following characteristics:

The instructor does not know "the answer", but must function as an expert problem solver side-by-side with students. Students gain confidence as experts themselves when they can problem-solve along with the teacher and not turn to the teacher as the ultimate authority and judge.

Students are asked to draw conclusions relying on their own data (their own experience). Students begin to see themselves as capable of constructing knowledge based on data collection and reasoning skills.

The results are communicated to others in the class. Although oral reports are intimidating, if the classroom environment is supportive, students can gain a voice through oral reports. Many women have trouble developing a voice and speaking persuasively. Structured oral reports may help. (Only if the environment is supportive. Otherwise, they may do more harm than good.)

The experimental task is within the intellectual ability of the students and there are adequate tools available to perform the experiment. Women students will quickly internalize failure (I can't do science) when the cause of the failure is faulty equipment or a problem that is too hard for them to solve. A little bit of failure goes a long way to convince girls that they can't do science. ❖

BioQuest Program Demonstration

Separate Knowing

"Presented with a proposition, separate knowers immediately look for something wrong—a loophole, a factual error, a logical contradiction, the omission of contrary evidence."

Separate Knowing in Science

Review of journal articles
The question and answer period at the end of a scientific presentation at a meeting
The analysis of data to make sure no interpretation has been left out
The design of experiments to insure that the experiment will answer the question being asked unambiguously

Connected Knowing

"When someone said something they disagreed with or disapproved of, their instinct was not to argue by to 'look at it from that person's point of view, see how they could say that, why they are right, why it makes sense.'"

Connected Knowing in Science

Defining and narrowing the question being asked
In communications about the set-up and performance of the experiment
In group analysis of data
In proof-reading papers
In lab meetings when newly formed ideas emerge

Other Readings

Carol Gilligan, *In a Different Voice.*
Sheila Tobias, *They're Not Dumb, They're Different: Stalking the Second Tier.*

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PART THREE

Additional Components of the Symposium

Questions & Reactions Generated by the Symposium Participants

Introduction

THROUGHOUT THE FIRST DAY of the Symposium participants were asked to react to each speaker by submitting a comment or question on a 3x5 card. The "one-minute reflections/questions" were collected and summarized to become a source of topics for the evening discussion sessions. These subjects included:

- Co-operative Learning
- Parent Involvement
- Target Students
- Teaching Styles and Strategies
- Learning Styles
- Self Esteem
- Single Sex vs. Coeducational Schools
- Journal Keeping
- Reaction Papers
- Preparing Kids for College
- Miscellaneous Questions and Observations

Co-Operative Learning

- Specific methods for setting up, monitoring, and using co-operative group learning in Math?
- What is the agenda for a single-sex school?
- Do any participants use the "study group" concept?
- What is the ideal size and personality make-up? How often?

- How do we work out the group assignments? — If the teacher makes the group assignments, girls tend to drift back to their social groups!
- How can competition and collaborative learning be seen as complementary to each other as well as opposed? Men are taught to share and work in teams, women are not.
- What is the role of athletics, team sports?
- How do you enforce group study (Harvard)?
- What to do when one member of a lab pair consistently does write good lab reports but is a weak student otherwise?

Parent Involvement:

- Let's talk about the influence of fathers on our girls.
- What can we or schools do to educate the parents about encouraging their daughters to be involved in math and science courses? Specifically boarding schools?
- Education of parents that it is good for girls to learn to be leaders, to learn to make their own decisions, to learn to take charge, to learn that their girls do not have to follow stereotype society has set for women.
- How do we respond to comments such as "I was never good at math, no wonder she's having trouble."?

Target Students:

- Would putting our target students in an all-girls' school also allow remaining students to fill the vacuum?
- I find that I do tend to call on "target students" more often. What can I do to be more aware of the other, quieter students?
- How do we control the more active, higher energy, disruptive students?
- It's the squeaky wheel syndrome, which excludes the "good" kids.
- How can you avoid the creation of target students?

Teaching Style and Strategies

- What are some ways to reduce apologetic questioning? To increase the ease of critical behavior?
- Balance competitive vs. collaborative, nurturing behavior so students are not shocked and discouraged when they arrive as an intended science major in college?
- How accepted is the University of Chicago math curriculum?

Learning Styles

There were questions of all sorts, from the origin and validity of the research to ways of making use of the conclusions.

Self-Esteem

- I am interested in knowing more about how competition in or out of the classroom affects students self-esteem.
- Should we encourage more competition in science fairs, math olympiads, sports, as a way of dealing with girls?
- In a coeducational setting, collaborative efforts sometimes force timid students to overcome inhibitions (but not necessarily in a healthy and positive way). Can this be more detrimental than what it is worth, particularly in the case of women and other minorities?
- If a student doesn't want to be involved, should we respect this?
- How might we be discriminating against the less vocal student?
- Do some students resent being called on by teachers and exposing their "ignorance"? I have had students cry when called on to answer.
- How do female science faculty gain more self-esteem when, to prove themselves, they have to do twice as much as the men?
- The image of the scientist as depicted in our schools. How many scientists are heads of schools? How many are in the administration?

Single Sex vs. Coeducational

- If women learn differently than men and we teach to that learning style, are we preparing them for a very different type of world outside of school?
- In particular, are the lessons learned in all girls' schools and the opportunities experienced truly carried over?
- How do we convince girls in an all girls' school of the problems they will face in society and scientific research?
- What are the advantages, disadvantages of single sex education for girls?
- Does the boarding experience for girls enhance the possibility of creating a culture in which math/science skills are fostered?

Preparing Kids for College

- At SUNY there is an especially successful math major program. Does anyone have more information on this? As understood, the program uses

study groups, group grading, collaboration. Why isn't this done more in colleges?

- How do we deal with the contradiction between collaborative learning and the SATs, Achievements, etc.—which will help them get into college?

Miscellaneous

Much of what happens to girls in society and in our schools is subtle and needs a lot of thought and attention to see it and make changes.

Evening Group Discussions

Some teachers wanted to know more about "equity training" mentioned by Kahle. A resource for this is Women's Action Alliance in New York City. Project SEED works with similar classroom equity issues at Marlborough School in Los Angeles. (contact Arlene Forsheit).

Preparing for College Majors

- How can we best prepare girls for the college science/math major? The two approaches mentioned by Jacobs and Kahle were discussed, the general consensus being that it is better to create an "oasis" so that skills and confidence can be built before sending students to college.
- How many women's colleges have an engineering major?

Resistant Parents

- How do you handle resistant parents—parents who don't want their daughters to take math? We found instead that parents are tending to push for more math and parents get nervous over average grades and performance. Students are under intense pressure to perform and have low self-esteem regardless of performance. One school has considered eliminating grades on tests.

Journals & Reaction Papers

- How do teachers feel about journals? How do teachers who use them organize the mechanics? Some do not grade it, and make it optional. One teacher gives specific mathematical topics or problems to be answered in words. One science teacher has them keep a journal as they build a project. Some use class time, some require them as homework, some make them due on test days. A suggestion was made that the

entire class keep one journal. Whatever the journal's form, its purpose must be clear to both teacher and student.

- What is the purpose of any journal? Can they be used for exploring issues such as women in science? One teacher uses them as a form of self-evaluation or teacher evaluation. Another teacher uses them to connect science to the student's daily lives. This can take the form of article reviews from newspapers, etc. Yet another has them write about a specific problem.
- Reaction papers can be used instead of journals to encourage writing. Journals are very much based on the *personality* of the teacher using them. Some forms will work for one teacher, but not for others. Reaction papers are short (1-2 minute) written responses to direct teacher questions. They can be used to evaluate student understanding of a specific idea or problem. No one, aside from Judy Jacobs, has used this approach.

Cooperative Learning

- Has anyone had good/bad experience with cooperative learning? Several teachers have tried it. Some assign study groups to operate inside and outside of class. Students do need to be taught how to do it. Maybe you should start with groups of two. There is no research consensus on whether homogeneous or heterogeneous groups work better. Groups need not be formally structured with individual tasks, but such structure will compel all to participate.
- How should we keep ourselves from creating target students, or once created, how should we handle it? It was suggested that a student be assigned to count who is called on and how often. Target students will always arise in any class, to a degree, but how do you draw out quieter kids? One teacher gives oral quizzes, another has each student explain problems on the board.
- Are there alternatives to the current way of evaluating students?
- How can we reduce students' obsession with grades? We can start by recognizing that we don't have to test everything we teach.

Parent Involvement

Parents need education, but that may take one or two generations before we begin to see results of our efforts. Parents have preset notions and attitudes towards daughters and curriculum expectations that are often unrealistic. At higher socio-economic level, teachers have less influence on girls than parents. Schools, teachers can educate parents at several levels:

- Use fall open house to talk to parents.

- Opening assembly with department heads speaking.
- School mailings.

Cooperative Learning

- Need to reconcile new techniques of teaching with parent expectations for SAT scores, etc.
- Time management problem - students turned on to math, but will have to sacrifice coverage of material.
- APs: convince parents they're not important? Want higher scores? Less time for cooperative learning.

Using Cooperative Learning

Can't just jump in - need to be prepared to keep groups on tasks and progressing. Promote group work outside class - on homework and related tasks. Students more familiar with cooperative learning will move into work more rapidly.

Assessment techniques for group work:

- Have preset standards for evaluation - can discuss and develop these with students.
- Group grade - small percentage of semester grade.
- Peer evaluation within group, including self-evaluation.
- Testing groups - groups begin to care less about evaluation?
- Give group improvement points if member improves?
- Testing and quizzes for individual students - this is competitive side of learning.

Dealing with Target Students

- Use wait time - no quick reaction to calling out or instant response.
- Be aware of which students are target students - may have to limit the number of times they may speak up in class.
- Be aware of quiet students as well.
- Difficulty with auditory learners—how can you let them verbalize?

Preparing Students for College Classes:

- Students who most benefit from cooperative learning often pressured to go to very competitive colleges.

- Try recreating collaborative learning situation by finding study group in college.
- All girl environments - more competitively inclined? May be preparing girls for competition of college - learning to take risks and fail.

Are Competitive Sports an Asset?

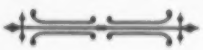
- Correlation between sports and competition in academic/professional environment?
- Concern that there may be too strong an emphasis on sports at present. Cuts large block of time out of day. Could be cyclical.
- Girls who are heavily programmed into sports tend to be more structured and better organized.
- Should we try to draw out the bright but very quiet girls?
- Depends on her confidence and focus? Self-esteem an issue - she may not be able to verbalize her competence and ability.
- We need to make girls aware of what to expect.
- Low level math classes often large and boring.
- May encounter unsupportive professors
- Stress to students - don't quit until you have completed at least two math courses - higher level classes smaller and more interesting.

Miscellaneous Questions & Observations

- Are girls less likely to drop out of math/science if they go to a small school?
- Girls will be used to small classes - need to develop the skills to survive in large ones.
- Parents less likely to complain when their daughter drops out of math/science than when their son does.
- Have feminists walked too far into the male camp in quest for change?
- Need to educate the boys too.
- Developing self-esteem is also a case of developing self-respect.
- Meeting responsibilities involved.
- Advantages of single-sex schools.
- In laboratory - boys tend to handle everything, girls shy away from equipment.
- Girls may act dumb in coed situations to protect social life.
- Girls often afraid to take a risk in coed situation.

Math and Science for Girls Follow-up Questionnaire

Site Visits



ON TUESDAY THE CONFERENCE PARTICIPANTS took to the road for on-site visits to the Mount Holyoke and Smith college intervention programs for secondary school girls. The first visit was to the Mount Holyoke College SummerMath Program. Dr. Charlene Morrow and her staff presented the philosophy of the SummerMath Program: "Emphasis on greater conceptual understanding, affirmation of young women as capable members of a learning community, and the importance of constructing one's own understanding of complex ideas." Participants were advised to emphasize process over outcome and conceptual understanding over answers. Participants entered into small group problem solving activities where they gained some understanding of their own relationship to the guiding precepts described during the brief presentation.

The afternoon visit to Smith College Summer Science Program was hosted by Dr. Beth Powell and her staff. This new program introduces a small number of secondary school girls to a hands-on research project. In its first year, the summer of 1990, students explored "Drugs and Behavior" using laboratory rats, "The Social Psychology of Prejudice and Discrimination" using interview research techniques, "Population Biology" using sampling techniques in the natural environment, and "Studies in Animal Development" using frog eggs and chick embryos.

Participants also visited the Wellesley College science and mathematics facilities. After a welcome by the Dean of Science, Wellesley faculty treated attendees to a hands-on activity in the new Wellesley Macintosh teaching laboratory. Fall 1991 Calculus classes will be taught using Mathematica in the lab. Participants also toured the science laboratory facilities, greenhouse, and observatory. Faculty members described the variety of research that their students pursue throughout the year. ❖

SummerMath

~ A mathematics learning community for young women in high school ~

by James Morrow

Co-Director, SummerMath, Mount Holyoke College

RESULTS OF THE 1988 NATIONAL ASSESSMENT of Educational Progress (NAEP) underscore the dismal state of mathematical understanding among Americans, especially in higher-level, problem-solving skills. The assessment finds that mathematical understanding is poorest for women, particularly minority women. Added to the NAEP's report on lack of mathematical skills is the astonishing and disturbing finding that an increasing number of women agree with the statement, "Mathematics is more for boys than for girls."

From much of what one reads, it may appear that most women are paralyzed by math anxiety. However, there are few sex differences to be found in mathematics course-taking at the secondary school level and approximately half of the bachelor's degrees awarded in mathematics go to women. Furthermore, math abilities are virtually the same for thirteen-year-old females and males. This is contrasted with the fact that by the end of high school boys are found, by most measures, to be better than girls at problem solving. It has also been found that many high school counselors still discourage women from taking advanced mathematics courses and from considering technological careers, that teachers interact more with their male students than their female students, that boys, far more than girls, utilize computer resources during unstructured time, and that parents still often consider mathematics, science, and technology to be more for males than females.

Such school, home, and social factors lead to a bleak career picture for women in science and engineering fields. As of 1984, women, who represent 43% of all professional workers, held only about 13% of the jobs in science and engineering. About 1% of engineers are women, 2% of physicists are women, and 5% of chemists are women.

The SummerMath Program

SummerMath was designed to address such underrepresentation of women in mathematics-based fields. It is an intensive, six-week program for females that provides new perspectives and new experiences of mathematics, computing, and science for young women. The program emphasizes greater conceptual understanding, affirmation of young women as capable members of a learning community, and the importance of constructing one's own understanding of complex ideas. Its instructional methods, based on a constructivist model of learning, are designed to remedy the blind memorization and rule-following behaviors that limit so many students' understanding of the underlying concepts of mathematics. SummerMath helps students replace such unproductive memorization methods with more flexible problem-solving approaches. The atmosphere of the program is one of challenge and support: the challenge of rigorous study and hard problems, and the support of a community of teachers, residential staff, and peers. The program's special approach to teaching helps students acquire greater confidence in their ability to excel. (See the article by Charlene Morrow for a more in-depth description of the pedagogical approach taken at SummerMath).

Classes

Students at SummerMath participate each day in classes characterized by student activity; questioning, discussion, and discovery. In all their classes students begin by solving problems or posing their own problems to solve. They experience a "new" vision of mathematics education through three classes: Fundamental Mathematical Concepts, Computer Programming, and Workshops.

The six-week course, Fundamental Mathematical Concepts (FMC), is the heart of SummerMath. Students are placed in FMC in one of three levels based on their academic preparation. They work and discuss ideas in pairs or in small groups, with at least one instructor and one undergraduate assistant circulating about the classroom asking probing questions and leading discussions. The curriculum focuses on concepts essential to understanding higher mathematics and science: ratio, linear relationships, functions, patterns, and logic. Students work with materials that present mathematics in progressive stages of development. They perform exercises that encourage them to take on the role of authority as they construct an understanding of fundamental concepts of mathematics. Problem solving is pervasive throughout all levels of instruction, and visualization and verbalization are stressed as means to deeper understanding. The nearly constant discussion among students and between students and their instructors provides a supportive environment, reflection on thinking, and the stimulus of collaborative effort.

The second major component of SummerMath is a course in computers and LOGO. Students work in pairs at a computer solving problems in the context of design projects. They pursue mathematical ideas ranging from elementary geometry to recursive functions. They learn to plan, organize, and revise their ideas by working on projects such as transformational geometry, tangram puzzles, patchwork quilt designs, and group murals. In the last week of SummerMath students demonstrate the programming skills they have developed by designing, developing, and completing a major computer project. In this collaborative and experimental environment, SummerMath students gain skill and confidence in computing.

Workshops comprise the third portion of the academic program. Each student takes three two-week Workshops, choosing from a variety of possibilities. There are Workshops in problem solving and confidence building, and there are Workshops in various scientific areas: Brain Imaging, The Art of Making Anatomical Comparisons, Applied Biology, Genetics, Economics, Psychology, and Physics of Motion. The scientific Workshops provide experiences that place students in the role of scientists—from thinking up the questions to be asked, to collecting data, to performing experiments and tests to compare what they have observed about the world with what they have conjectured about the world.

The Residential Community

The residential program is also a critical aspect of SummerMath. It helps create a strong and supportive community. Undergraduate residential assistants who are also teaching assistants, give participants support and friendship as they face the challenges of learning ideas in depth.

Students are offered many additional activities intended to link their math experiences to the outside world, to give them a realistic view of career options, and to broaden their horizons. Regular activities, including sports and recreation, trips and cultural events, are scheduled throughout the summer. Once a week, speakers representing science, business, and various professions present an evening lecture, panel discussion, or laboratory demonstration, and answer questions about their work and the relevance of mathematics to it.

Outcomes and Assessment

The most dramatic improvements are illustrated in students' comments on their own abilities to learn mathematics. These comments indicate that the students now find mathematics more challenging and more creative and see it as more than just memorizing formulas. Many have said they were more flexible and likely to be more persistent in solving a problem.

For the assessment of students, SummerMath uses tools that further the

educational process. There are ongoing discussions with faculty in meetings conducted during the program itself. Students evaluate their own progress at mid-program and discuss their assessment of themselves with their instructors. At the end of SummerMath there is a two-way evaluation done, with teachers commenting on the progress of the students in their classes and making suggestions for further work and students evaluating their classes: mathematics, computing, and each of their three workshops. During the academic year following the program students complete a questionnaire concerning changes in attitude, courses taken, new ways of approaching learning, and grades and standardized test scores.

Staff Preparation

The quality of the experience for students is assured not only by the considerable continuity and experience of the teaching staff, but also by an intensive week of staff preparation each year. This week is the basis for a strong community committed to the constructivist principles of active learning. The training week includes the development of goals, extensive practice teaching, the training of teaching assistants, and workshops on issues of importance to adolescents.

At SummerMath the process of solving problems and exploring ideas in small groups using computers, science laboratories, the outdoor environment, and pencil-and-paper develops a deep and secure understanding of fundamental ideas, how they developed, and how they are used. Learning a new approach to mathematics that emphasizes reasoning, becoming comfortable with the subject, and being motivated to study it further are the main outcomes of SummerMath for students. An additional benefit is a much clearer sense of what a college environment is like and what kind of career choices may lie ahead. These insights are particularly valuable to low-income and minority students, who often hesitate to consider college at all, but particularly in fields related to mathematics or science. This challenging and supportive program has proved extremely effective in furthering the mathematical and career aspirations of students from widely varying backgrounds, not just the mathematically precocious. ❖

***James Morrow** is currently Co-Director of the SummerMath program at Mount Holyoke College and Lecturer in Mathematics. He is also Executive Director of Women and Mathematics Education. His research interests include investigating the effects of a constructivist mathematics program for students and surveying attitudes of minority students toward mathematics. He holds a Ph.D. in Mathematics from Florida State University.*

Ideas, not Answers

~ *A process-oriented approach to mathematics learning* ~

Charlene Morrow

Co-Director, SummerMath, Mount Holyoke College

FOR THE PAST NINE YEARS, the SummerMath program at Mount Holyoke College has been engaged in the development of innovative approaches to mathematics education. (See the article by James Morrow p. 97 in this publication for a more general description of the SummerMath program.) Our methods are based on a constructivist pedagogy, but are also informed by research on gender differences in mathematics education. Our student population is comprised entirely of young women in high school. Our theory and experience has led us to emphasize process over outcome and conceptual understanding over answers. At present it seems that the main forces in mathematics education are converging toward these same goals, as best represented by the National Council of Teachers of Mathematics' *Curriculum and Evaluation Standards* and forthcoming *Professional and Teaching Standards*.

Outlined below are the major themes we draw from constructivist pedagogy and current gender research in mathematics education. This discussion is followed by examples of our teaching methods, which are designed specifically to create a constructivist mathematics classroom that best serves the needs of young women.

Constructivist Pedagogy

The foundation of this pedagogy is the belief that people come to understand mathematical concepts through personal experience. Activity of the student and focus on understanding rather than rote learning are emphasized in constructivist mathematics classrooms. The teacher's role becomes that of guide and mentor rather than knowledge-giver. The curriculum is designed to maximize the student's opportunities to fully understand each concept through problem solving experiences that are developmental with respect to that concept. Students come to see relationships between what is known and what is questioned, thereby extending present conceptual

understanding. Specific mathematical material and effective ways of approaching new material are equally important in a constructivist learning environment.

Gender Themes in Mathematics Education

Over the past twenty years a great deal has been written about the ways in which traditional mathematics classes have served women poorly, leading to low participation rates of women in mathematically related fields. In attempting to make mathematics instruction effective for young women, our classes take into account the following findings:

- Females prefer an environment where students are not working alone and in isolation.
- Knowing the utility of mathematics and its connectedness to other areas seems critically important to females.
- Females are frequently allowed to be more passive than males in school and are less often asked to develop justifications for their ideas.
- Opportunities to explore the world spatially occur less often for females than males.
- Feeling confident in mathematical ability is more predictive of course-taking for females than males.
- The computer culture outside of the classroom is predominantly male.

SummerMath Classroom Methods

Answers to problems are not given by the teacher. In order to help students begin to gain a sense of their own knowledge base in mathematics, we first need to help students move away from a strictly answer-oriented approach to solving problems. We have found that it is impossible to achieve this goal in an environment where answers are supplied, therefore, we simply do not tell students whether they are right or wrong. Nor are students told how to solve a problem. When neither answers nor instructions are supplied, students are forced to look to themselves and to each other in order to discover what they might already know about the problem. They must also devise an approach to solving the problem.

Decreasing answer-dependency helps to accomplish several goals. First, students are reconnected with their intuitive bases and will increasingly be able to draw on this knowledge base. Second, the excitement for a student of discovering that she can solve a problem that she initially did not know how to do contributes significantly to the motivational base needed to persist in mathematics learning. Finally, solid conceptual development in mathematics occurs only through the student herself thinking and working through problems, not solely through the observation of others' problem solving processes.

The Teacher Acting as Facilitator & Guide

Obviously answers are not simply withheld from students without other support systems being in place. The teacher's role is critical, not only in terms of providing information, but in facilitating the problem solving process so that when information is provided, it is on the student's own terms. The teacher must become skilled in active listening and questioning that will allow the student to become more aware of her own thinking and to decide which ideas to pursue further. Active listening by the teacher can more quickly allow misconceptions to surface and be corrected. It is also much clearer when conceptual understanding has been attained.

This kind of teacher role has been likened to the Socratic teacher-student relationships; however, I would like to point out a critical difference between the approach being discussed here and the Socratic method. As demonstrated by Socrates himself, the Socratic teacher maintains control of the dialogue mainly by asking leading questions and defining the terms of the conversations. In a constructivist classroom, the student is in charge of the interaction. The direction of talk is mainly defined by the student. The teacher, of course, will inevitably provide a "safety net" whereby misconceptions are brought to the surface, recognized, and corrected.

The balance between not asking leading questions, but keeping students from going too far down a non-productive track is delicate and develops with experience.

Students Interact Mainly in Pairs and Small Groups

In order to maximize the time that students are actively engaged in problem solving and conceptual development they work in pairs in all parts of the program, including computer programming. In the beginning each student in the pair is given a specific role: one is the problem-solver and the other is the active-listener/questioner in the same manner as described above under teacher role.

Students switch roles as they move from one problem to the next. As they become more experienced and comfortable with both roles, these roles begin to merge into a cooperative problem solving experience. Students become better both at persisting in problem solving and in asking themselves questions that will help them to move along. They also become better at asking specific and focused questions of teachers when help is needed.

As students solve problems in pairs, they are encouraged to turn to other pairs to compare solutions. Whole class discussions usually occur at two particular points. First, when a new concept is being introduced students are encouraged to think about what they already know about this area. Second, when a problem has been solved by all students each is asked to put her solution on the board, then solutions are explained by each student,

compared, and discussed. This process serves to highlight the varied ways in which a problem can be solved.

Students to Represent Solutions Pictorially

One of the best ways we have found to get students in touch with their existing knowledge base in mathematics and to facilitate alternative paths to solving a problem is to have students represent their processes and solutions pictorially. Students report that being able to "see what I am doing" is a very powerful experience. In the process of learning and using the computer language LOGO in the computer class, students' ability to visualize mathematical ideas is also extended and deepened. They become much more effective and confident as their thinking becomes evident.

Implicit in the methods discussed above is the notion that in order to arrive at a solution, a student must be able to explain the process by which she came to an understanding of the problem. Unless a student can demonstrate to herself that she knows why she has solved a problem, she will remain unconfident and teacher-dependent. Justifying answers to yourself is a very powerful tool in mathematical understanding.

Mathematical connections in areas outside of mathematics are emphasized. Workshops in mathematically related fields such as genetics, chemistry, and medicine help students learn about the ways in which mathematics is applied in a variety of settings. Writing exercises also provide an effective mechanism for connecting mathematics to other realms and for contextualizing mathematical knowledge. As mentioned above, understanding the place and utility of mathematics is an important factor in the persistence of young women in mathematical studies.

As students engage in mathematics classrooms based on the principles discussed above, they become much more able problem solvers, ready to approach problems flexibly, confidently, and persistently. They become independent, but not isolated problem solvers, retaining a sense of authority within themselves, but able to communicate effectively with others about mathematics. ❖

***CHARLENE MORRIS** is currently Co-Director of the SummerMath program at Mount Holyoke College and Lecturer in Psychology and Education. She is also the President-Elect of Women and Mathematics Education. Her research interests include investigating women's ways of knowing in mathematics and minority students' attitudes toward mathematics. She holds a Ph.D. in Clinical Psychology from Florida State University.*

Smith Summer Science Program

~ A scientific research program for high-school girls interested in psychology and biology ~

Beth Powell

Opening Comments

THE SUMMER SCIENCE PROGRAM sponsored by Smith College is a rigorous four-week program designed for young women who are interested in learning more about scientific research in the areas of psychology and biology. Students participate in two challenging research programs conducted by Smith College faculty in modern research laboratories and have the opportunity to design and conduct experiments on such diverse topics as discrimination and prejudice, drugs and behavior, developmental biology and evolution. In addition, students are encouraged to participate in a variety of career-planning, recreational, and cultural activities.

Courses and Faculty

Psychology: Discrimination and Prejudice
Taught by Fletcher Blanchard

This course will explore the current research literature on the contemporary social problem of the unintentional expression of discriminatory behavior by persons who believe themselves to be nonprejudiced. Students will design and conduct original research projects and opinion polls concerning racial prejudice and procedures that promote interracial respect and friendship. Students will learn to use microcomputers to write research reports and to analyze the results of their research projects.

Psychology: Drugs and Behavior
Taught by Beth Powell

There is a persistent belief in our society that alcohol is not a drug. In order to dispel this myth, students in this course will examine the literature on

alcohol's effects on human and animal behavior. Students will confront the difficulties of studying this problem in humans and will develop an animal model of alcohol abuse. The effects of alcohol on a variety of behaviors in rats and mice will be examined. Particular emphasis will be placed on sexual and maternal behaviors. The course will also address the ethics and value of using animals in experiments designed to understand human problems.

Developmental Biology
Taught by Jeanne Powell

Students will study several developing systems, such as the heart and limb, and design experiments to explore the control mechanisms involved in development and/or regeneration. Techniques of microdissection, organ and tissue culture, and scanning electron microscopy will be used.

Population Biology
Taught by Steve Tilley

Populations of organisms are living systems which change in size over time. They are vast storehouses of variety, the foundation of evolutionary changes. Evolution, in fact, happens to populations, not organisms. This course will demonstrate what populations are, why they are interesting, and how methods developed by ecologists, computer scientists, biochemists, and mathematicians can be used to study them. Using the human population as an example and employing computer simulations, we will see how a population's size changes and how rates of population growth are measured. We will then go into the field and compare different populations of a local amphibian species, the red-spotted newt, to see how the sizes and other features of natural populations can be estimated and compared. Finally, we will examine some of the genetic properties of populations of red-backed salamanders, using a biochemical technique called electrophoresis. ✦

Teachers Teaching Teachers

~ *The Creation of the Task Force Reports* ~

THE CULMINATING ACTIVITY OF THE CONFERENCE was the collaborative process of sharing and expanding the participants' own wealth of teaching experience. The process began with an individual writing exercise. Each participant was asked to describe in detail one classroom activity that worked particularly well for girls. Four discipline areas were formed: MATHEMATICS, MATHEMATICS AND TECHNOLOGY, PHYSICAL SCIENCE, AND BIOLOGY. Individuals shared the results of their writing exercises with the group, and lists of activities, strategies, and methods began to grow on blackboards throughout the conference space. Groups struggled to give names to the related subjects, and small subgroups were formed to discuss the topics which evolved—to bring shape to them, to connect them to the research shared earlier in the conference and finally to prepare written and oral reports for the finale of the symposium, the Task Force Reports. These Reports have been published separately and can be ordered by writing: The National Coalition of Girls' Schools, 228 Main Street, Concord, Massachusetts 01742. (508) 369-1484 • *Fax* (508) 369-3452. ❖

Symposium Participants

~ The lesson plans, shared philosophies, and challenges to classroom routines are the contributions of the following symposium participants ~

Itir Arkan, *Marymount School*
Hilda Bachrach, *Dana Hall School*
Barbara Bass, *St. Catherine's School*
Amy Berner, *Annie Wright School*
Cornelia Brunner, *Speaker*
Dorothy Buerk, *Speaker*
Melby Bush, *Maderia School*
Paulette Cantey, *Ashley Hall*
Ann Charlson, *Hockaday School*
Elizabeth Chesick, *Baldwin School*
Jennifer Christy, *Miss Porter's School*
Linda Clayton, *Dana Hall School*
Anne Considine, *Marymount School*
Eileen Core, *Kent Place School*
Sharon Corrigan, *Westover School*
Kay Cortelyou, *Maderia School*
Joan Countryman, *Speaker*
Patricia Couts, *The Holton-Arms School*
Sally DeMott, *The Maderia School*
Lois DePolo, *Taft School*
Bill Doswell, *Miss Hall's School*
Beatrice Doyle, *White Mountain School*
Monica Edwards, *The Masters School*
Deborah Fischer, *Garrison Forest School*
Arleen Forsheit, *Marlborough School*
Charlotte Freeman, *Girls' Preparatory School*
Rae Friesen, *Academy of New Church*

Marilyn Gilbert, *Kent Place School*
Louise Gould, *Ethel Walker School*
Marie Gransbury, *Miss Porter's School*
Phyllis Gwatkin, *Hopkins School*
Nancy Habenicht, *St. Catherine's School*
Grant Hagerman, *Bishop Strachan School*
Michael Hallaran, *Westover School*
Lee-Ann Harris, *Ethel Walker School*
Claudia Henrion, *Speaker*
Elisabeth Holloway, *Hockaday School*
Margaret Honey, *Speaker*
Carolyn Hopp, *Speaker*
Carol Hubbard, *Speaker*
Kanta Jayachandran, *Santa Catalina School*
Steve Jenkins, *Dana Hall School*
Francine Johnson, *St. Paul's School*
Julie Keeling, *Groton School*
Terry Keller, *Foxcroft School*
Judy Kemlitz, *Garrison Forest School*
Linda Klett, *San Domenico School*
Rosa Kuppermann, *Westridge School*
Cynthia LaMothe, *Winsor School*
Susan Layard, *Bishop Strachan School*
Barbara Lazarus, *Speaker*
Christine Lind, *Westover School*
Pauline Lopez, *Kent Place School*
Jan Maguire, *Nashoba Brooks School*
Susan Malone, *Mercersburg Academy*
Al Mann, *Miss Porter's School*
Keith McBride, *Foxcroft School*
Linda McIntosh, *Dana Hall School*
Finlay McQuade, *Consultant*
Margaret McQuillan-Burns, *Miss Porter's School*
Binney Miller, *Kent Place School*
Eric Mulfinger, *Westridge School*
Jane Newberry, *Beaver Country Day School*
Patricia Nolan, *Taft School*
Judy Nuno, *Georgetown Visitation School*
William Page, *Newton Country Day School*
Robert Patrizi, *Hockaday School*
Nina Patterson, *Choate-Rosemary Hall School*

Ann Pogrel, *San Domenico School*
Ann Pollina, *Westover School*
Caroline Reed, *Hewitt School*
Pat Roach, *Georgetown Visitation School*
Florence Rodgers, *Chapin School*
Janice Rogers, *Emma Willard School*
Linda Samuels, *Dana Hall School*
Harriet Scheuermann, *National Cathedral School*
Bill Schirmer, *Dana Hall School*
Katharine Schmitt, *Concord Academy*
Jan Serie, *Speaker*
Charlotte Sheridan, *Ashley Hall School*
Kathleen Skelly, *Dana Hall School*
Caroline Smith, *St. Mary's School*
Neil Smith, *St. Andrew's School*
Rebecca Steinbach, *Andrews School*
Barbara Steinhoff, *Dana Hall School*
Pamela Stengel, *Chatham Hall School*
Jinx Tong, *Vivian Webb School*
Stephanie Weber, *Purnell School*
Virginia Whitehead, *Chatham Hall*
James Williams, *Emma Willard School*
Rita Wilson, *Girls' Preparatory School*

The National Coalition of Girls' Schools

THE NATIONAL COALITION OF GIRLS' SCHOOLS is a collaborative of sixty-five girls' boarding and day, public and private schools from across the United States. While differing in history, tradition, location, academic rigor, extracurriculars, religious affiliations, size, and philosophies, Coalition members share a belief in and commitment to the values and advantages of an all-girls school. To that end, the organization has collected research on gender issues; surveyed member school alumnae, parents, and prospective families; sponsored national forums for leading girls' and womens' groups; gathered comparative institutional data; and developed newsletters profiling leaders and achievers at girls' schools. ***The Math and Science for Girls' Symposium*** was the first in what is hoped will be a series of timely conference offerings. ❖

For More Information

About the Coalition, its Publications,
Research, and/or Directory
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